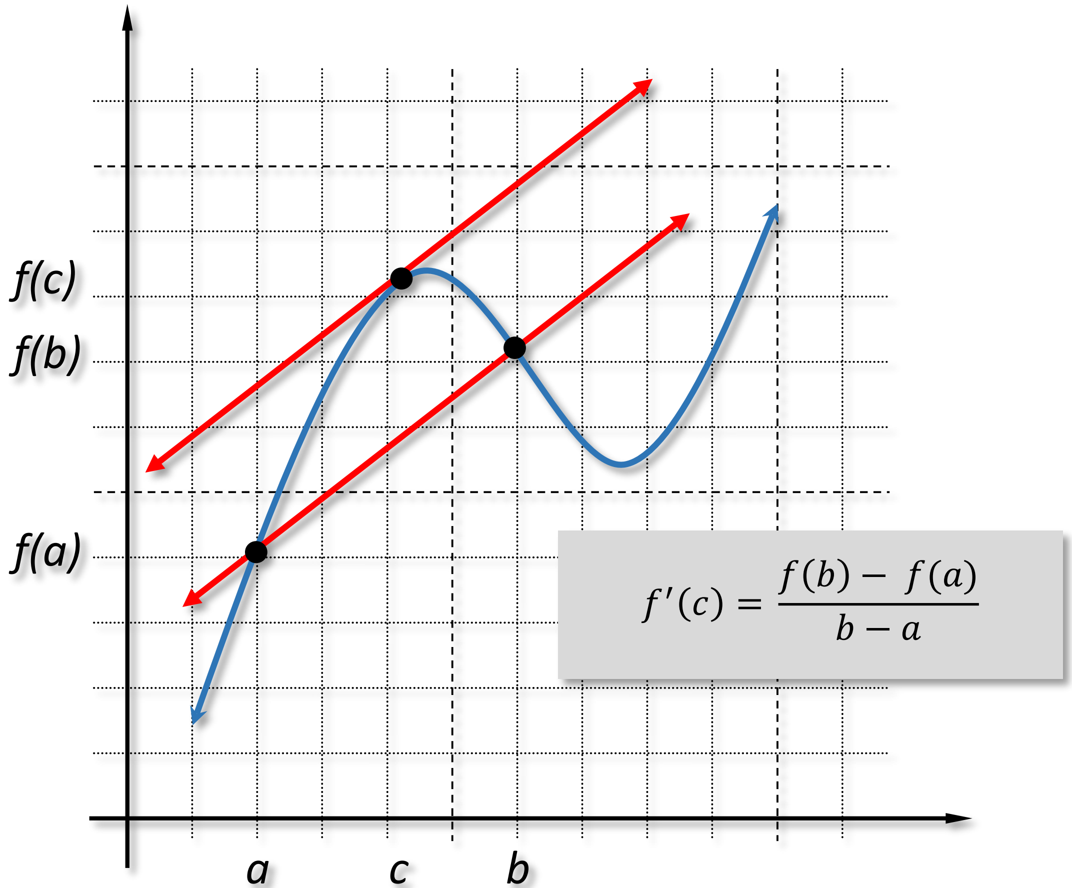


Name ________________________________

UNIT 5:

ANALYTICAL APPLICATIONS OF DIFFERENTIATION HOMEWORK

5.1 The Mean Value Theorem Homework

Problems 1 – 4, Determine whether Rolle's Theorem can be applied to the function on the closed interval. If Rolle's Theorem can be applied, find all values of $x = c$ that satisfy the theorem, or explain why not.

1. $f(x) = x^2 - 3x - 10 \qquad [-2, 5]$

2. $f(x) = x^{2/3} - 2 \qquad [-1, 1]$

3. $f(x) = \sin x \qquad [0, 2\pi]$

4. $f(x) = x - 2\ln x \qquad [1, 3]$

Problems 5 – 10, Determine whether the Mean Value Theorem can be applied to the function on the closed interval. If the Mean Value Theorem can be applied, find all values of $x = c$ that satisfy the theorem, or explain why not.

5. $f(x) = x^3 - x^2 - 2x \qquad [-1, 1]$

6. $f(x) = \sqrt{3 - x} \qquad [-6, 3]$

7. $f(x) = \frac{x+3}{x}$ $\left[\frac{1}{2}, 3\right]$

8. $f(x) = |2x - 1|$ $[-2, 3]$

9. $f(x) = x^3$ $[-1, 0]$

10. $f(x) = x \ln x$ $[1, 2]$

11. Mr. O. ran the Disney Marathon in 2 hours 18 minutes. Remember a marathon is 26.2 miles. Show that at least twice he had to be running at a rate of 11 miles per hour.

12. Cecil's *ABC Cars* sells used automobiles. His sales, for the year, S, are a function of time, in months, given by $S(t) = 240\left(6 - \frac{8}{t+2}\right)$. Find the average rate of cars sold over the entire year for $0 \le t \le 12$. During what month does the average rate of car sales equal the rate of change of the car sales?

13. Contaminated water is being pumped from a retention pond. *R(t)* is the amount of water that is left in the pond at time *t,* measured in hours.

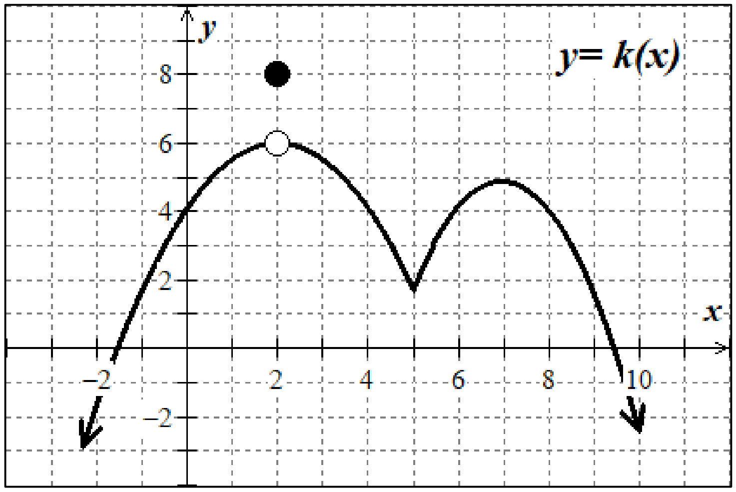
t, *hours*	0	3	6	9	12	15	18	21	24
$R(t)$ *gallons*	20,000	17,000	16,000	14,500	13,000	10,000	9,000	7,000	5,600

A. Using correct units of measure, find the average rate of change of $R(t)$ from $t = 3$ hours to $t = 21$ hours.

B. Estimate the rate that water is being pumped at $t = 8$ hours. Use the correct units. Explain what this value means in the context of the problem.

C. Is there a time on $0 < t < 24$, such that $R'(t)$ must be equal to $-600\ gal/hr$? Justify your answer.

Problems 14- 17, Given the function $y = k(x)$ shown below, discuss whether Rolle's Theorem or the Mean Value Theorem can be applied on the stated intervals. Justify your reason.

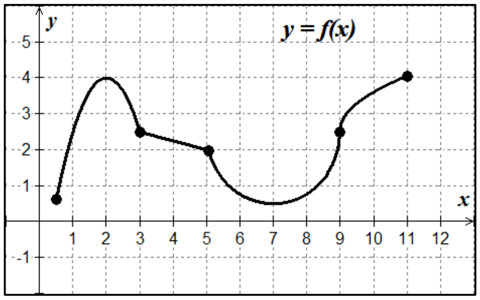


14. $[5, 8]$ **Mean Value Theorem**	
15. $[0, 4]$ **Rolle's Theorem**	
16. $[3, 6]$ **Mean Value Theorem**	
17. $[4, 8]$ **Rolle's Theorem**	

5.2 Extrema on an Interval Homework

Problems 1 – 4, use the graph to estimate the absolute extrema and local extrema.

1.



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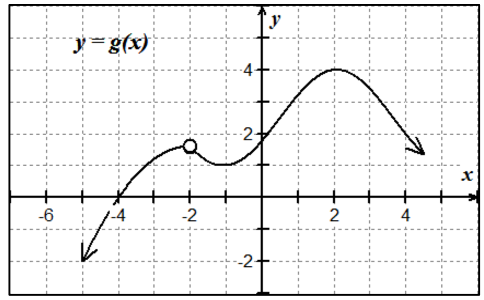

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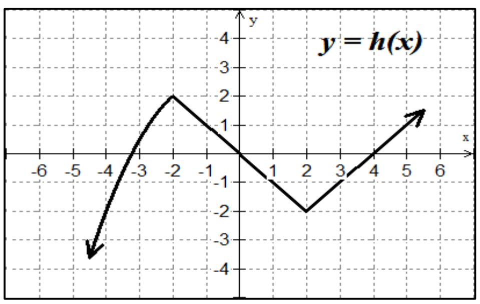

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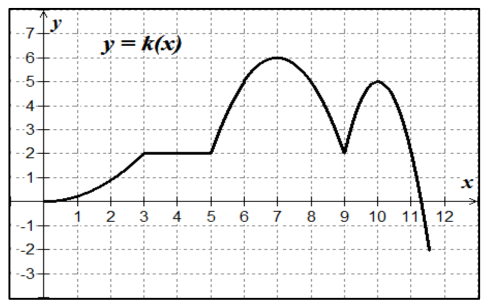


Problems 5 – 10, determine the critical numbers for each function below.

5. $f(x) = \ln\left(x^2 - 4\right)$

6. $h(x) = \sqrt{x^2 - 9}$

7. $y = 2x^3 + x^2 - 20x + 5$

8. $s(t) = t^2\sqrt[3]{2t - 5}$

9. $g(x) = \frac{2x-3}{x^2-9}$

10. $h(x) = x \cdot ln\, x$

Problems 11 – 16, find the absolute maximum and minimum values of *f* on the given closed intervals and state the critical numbers where these values occur. Give decimal approximations when prompted by calculator icon.

11. $y = x^3 - 6x^2 + 8$ $\quad [1, 6]$

12. $y = x^3 + x^2 - x$ $\quad [-3, 3]$

13. $f(\theta) = \sqrt{2}\theta - \sec\theta$ $\left[0, \frac{\pi}{3}\right]$

14. $y = 3e^x - e^{2x}$ $\left[-\frac{1}{2}, 1\right]$

15. $y = \frac{\ln x}{x}$ $[1, 3]$

16. $f(x) = \left(x^2 - 3\right)^{\frac{2}{3}}$ $[-3, 2]$

5.3 Increasing, Decreasing and the First Derivative Test Homework

Problems 1-4; Identify the open intervals on which the function is increasing or decreasing.

1. $f(x) = 8x - x^3$

2. $g(x) = x\sqrt{9 - x^2}$

3. $h(x) = x + 2\sin x \qquad [0, 2\pi]$

4. $f(x) = e^{-x} + e^{2x}$

Problems 5 – 9; Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. Check your results with a graphing utility.

5. $f(x) = x^3 - 6x^2 + 15$

6. $f(x) = \frac{x^3}{3} - \ln x$

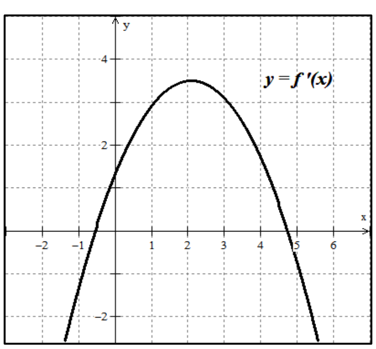
7. $f(x) = \sqrt[3]{x} - 1$

8. $f(x) = \frac{x^2}{x+1}$

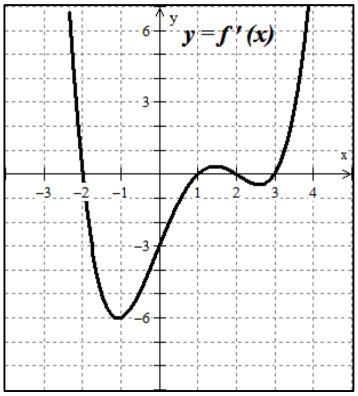
9. $f(x) = (x-2)e^x$

Problems 10- 11; Use the graph of f' to identify the interval(s) on which f is increasing or decreasing, and estimate the value(s) of x at which f has a relative maximum or minimum.

10.

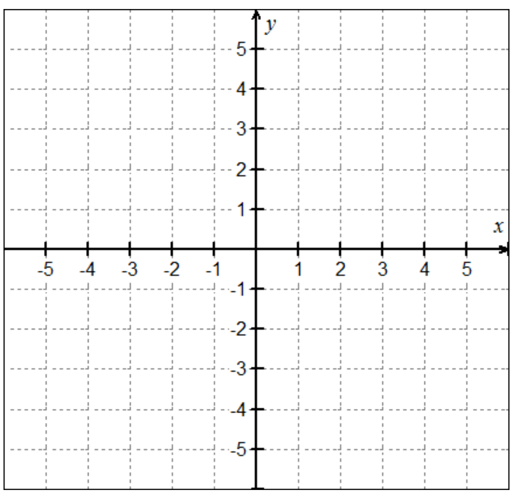


11.

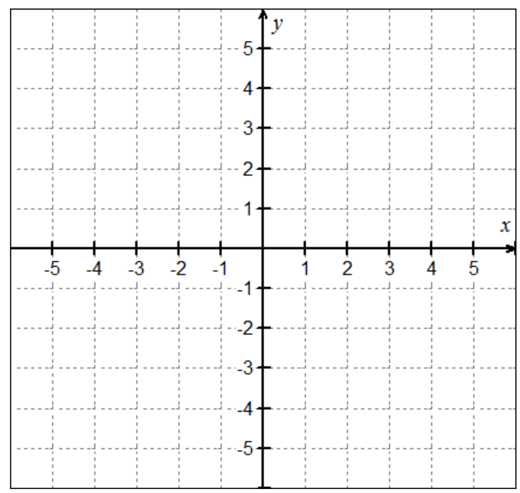


Problems 12 – 13; Sketch the graph of a continuous function f that satisfies all of the stated conditions.

12. $f(0) = 1$; $f(2) = 3$; $f'(0) = f'(2) = 0$;
$f'(x) < 0$ on $(-\infty, 0) \cup (2, \infty)$
$f'(x) > 0$ on $(0, 2)$



13. $f(1) = f(5) = 0$; $f'(3)$ does not exist
$f'(x) < 0$ if $x < 3$; $f'(x) > 0$ if $x > 3$
$f(x)$ is concave down if $x \neq 3$



14. Assume f is a differentiable function, where the signs of $f'(x)$ are as follows:

$$f'(x) > 0 \text{ on } (-\infty, -5) \cup (4, \infty) \text{ and } f'(x) < 0 \text{ on } (-5, 4).$$

Let $g(x)$ be a transformation of $f(x)$. Complete the inequality $(>, \geq, <, \leq)$ for the value of $x = c$:

Function	**Sign of $g'(c)$**
A) $g(x) = -f(x)$	$g'(-8)\ \square\ 0$
B) $g(x) = f(x - 8)$	$g'(1)\ \square\ 0$
C) $g(x) = f(2x) - 4$	$g'(-4)\ \square\ 0$

Problems 15 – 18: use the First Derivative Test to complete the following.

---

15. For what values of x is the function decreasing? $f(x) = 16 + 24x + 3x^2 - x^3$

A. $-2 < x < 4$

B. $-4 < x < 2$

C. $x < -2$ or $x > 4$

D. $x < -4$ or $x > 2$

---

16. Let $f(x) = x\sqrt{4 - x^2}$. Find all relative extrema of f.

---

17. Let $g(x) = 2xe^{-x}$; for $x \geq 0$.

A. Find where g is increasing and decreasing, Justify.

B. Determine the extrema of $g(x)$ on $[0, \infty)$. Justify.

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18. Find the relative extrema of the function $h(x) = x^2 e^{1/x}$. Justify.

5.4 Concavity and the Second Derivative Test Homework

Problems, 1 – 4, Determine the open intervals on which the function is concave up or down.

1. $f(x) = x^3 - 6x^2 + 4$

2. $g(x) = 2x^2 + \ln x\,; x > 0$

3. $h(x) = e^{-x^2}$

4. $f(x) = \frac{x}{x^2 + 1}$

Problems 5 – 10, Find the points of inflection and discuss the concavity of the graph of the function.

5. $f(x) = 2x^4 - 8x + 1$

6. $g(x) = x\sqrt{9 - x}$

7. $f(x) = x \, ln \, x$

8. $g(x) = xe^{-x}$

9. $h(x) = \sin x + \cos x \quad [0, 2\pi]$

10. $f(x) = \frac{4x}{x^2 + 1}$

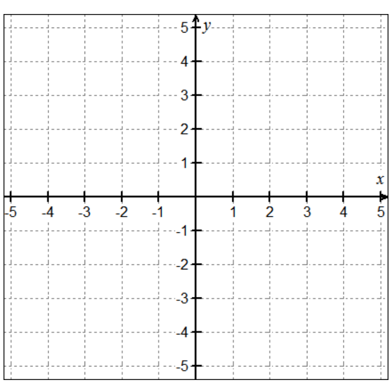
Problems 11 – 12, Sketch the graph of a function *f* having the given characteristics.

11. $f(0) = f(3) = 0$

$f'(x) > 0$ if $x < 1$

$f'(x) < 0$ if $x > 1$

$f''(x) < 0$



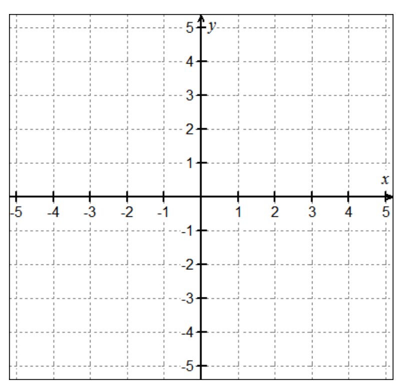
12. $f(-2) = f(4) = 0$

$f'(2)$ does not exist

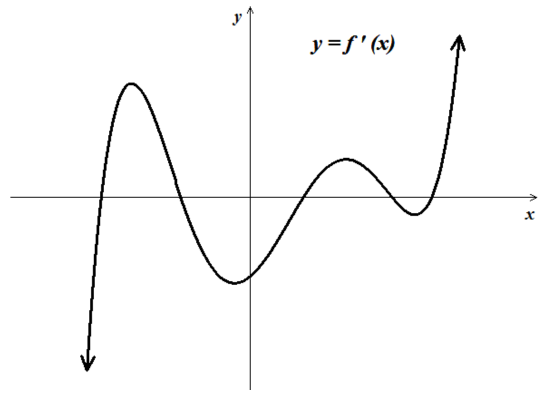
$f'(x) > 0$ if $x < 2$

$f'(x) < 0$ if $x > 2$

$f''(x) > 0 \; x \neq 2$



13. The figure at right, shows the graph of the derivative of a function, f . How many points of inflection does f have in the interval shown. Justify how you know.



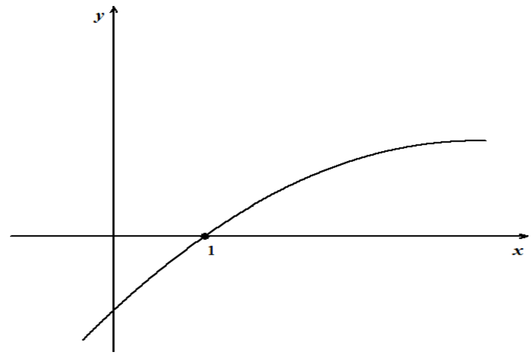
14. The graph of a twice-differentiable function f is shown at right. Which of the following is true?

A. $f(1) < f'(1) < f''(1)$

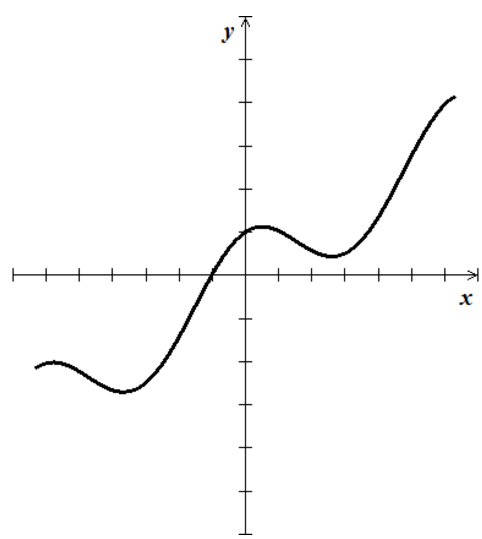
B. $f(1) < f''(1) < f'(1)$

C. $f'(1) < f(1) < f''(1)$

D. $f''(1) < f(1) < f'(1)$



15. The figure at right is a graph of the equation $y = \frac{1}{2}x + \cos x$ on the interval $-2\pi \leq x \leq 2\pi$. Use the second derivative test to find the local extrema of f.



Problems 17 – 18, Use the Second Derivative Test to find the local extrema for the given function. Show the analysis that leads to your conclusion.

17. $f(x) = -x^3 + 6x + 4$

18. $g(x) = \frac{x^3}{3} - x^2 - 3x$

19. Putting it all together. Given $f(x) = \frac{x^2}{x^2 - 1}$; Analyze the function and justify your answers.

A. Find the intervals on which f is increasing or decreasing.

B. The x-values of the extrema of the function.

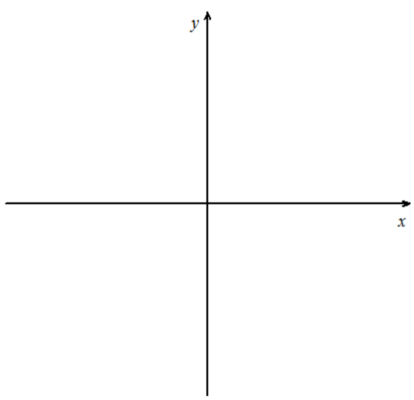
C. The points of inflection.

D. The intervals of concavity

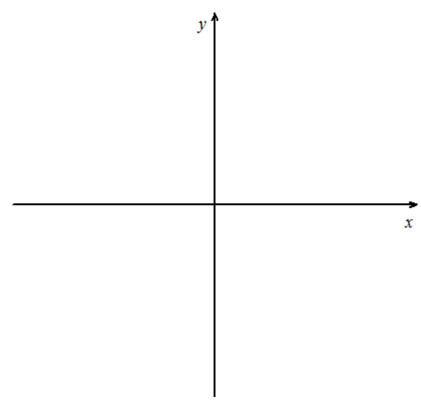
5.5 Curve Sketching and Connecting f, f', f'' Homework

Problems 1 – 4, Analyze and sketch a graph of the function. Label any intercepts, relative extrema, asymptotes, and points of inflection.

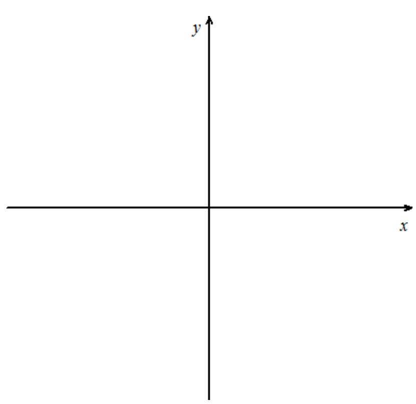
1. $y = x - 4\sqrt{x}$



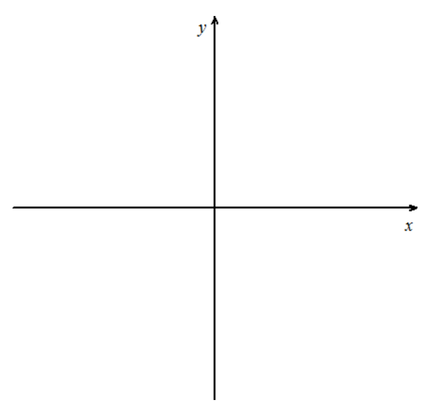
2. $y = \frac{x-3}{x}$



3. $y = 2\cos x - x$; $[0, 2\pi]$

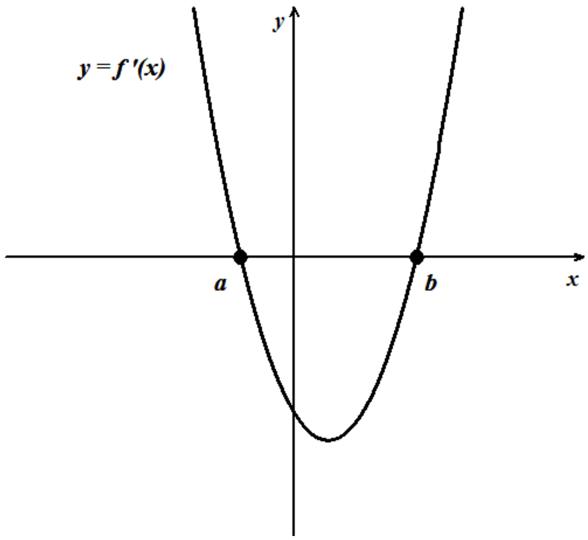


4. $y = x - 2\ln x$



Problems 5 – 6, Use the graph of the f' to sketch the graph of f and the graph of f''.

5.



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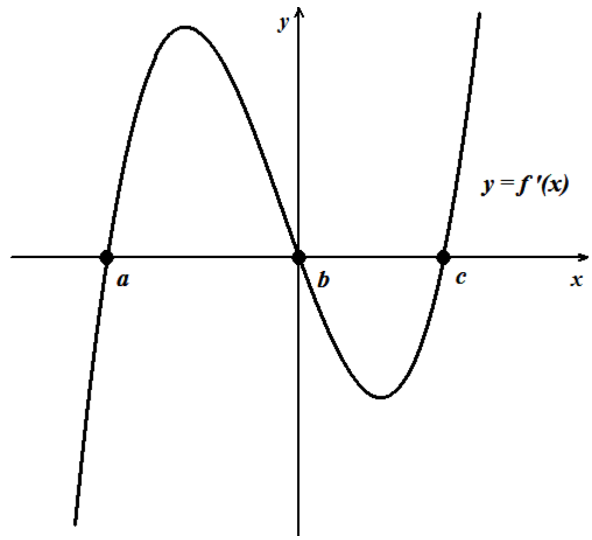
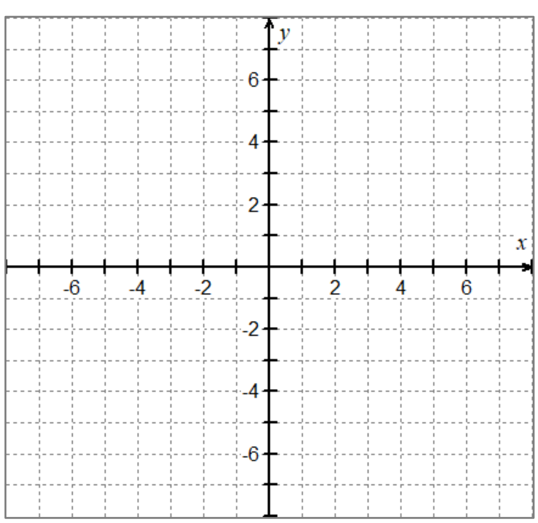


Problems 7 – 8, Sketch the graph of the function, f, given the following information.

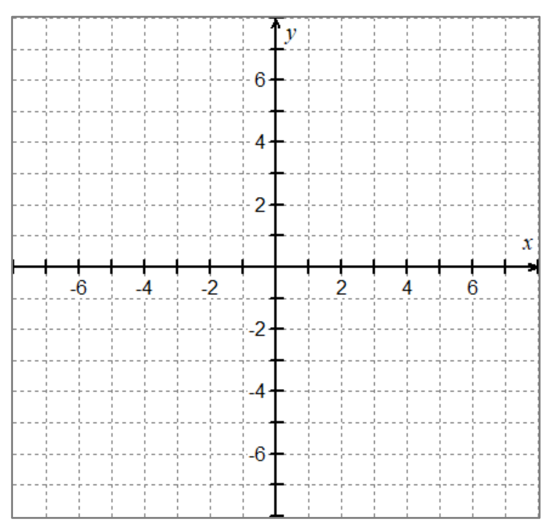
7.

A. continuous for all real numbers
B. $f'(x) < 0$ on $(-\infty, -3)$ and $(0, 3)$
C. $f'(x) > 0$ on $(-3, 0)$ and $(3, \infty)$
D. $f''(x) > 0$ on $(-\infty, 0)$ and $(0,6)$
E. $f''(x) < 0$ on $(6, \infty)$
F. $f'(0)$ does not exist



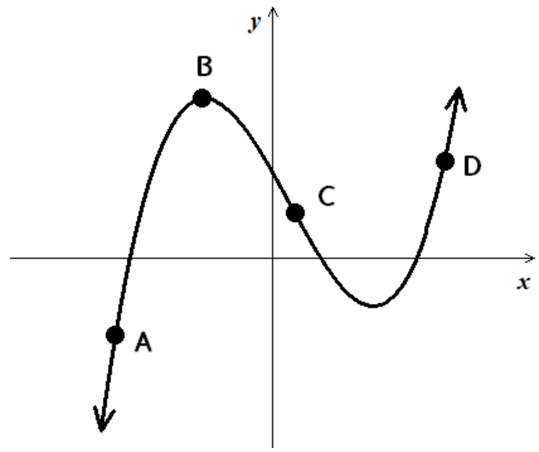
8.

A. continuous for all real numbers
B. $f'(2)$ does not exist
C. $\lim_{x \to -\infty} f(x) = 2$
D. $f'(x) > 0$ on $(-\infty, -1)$ and $(2,5)$
E. $f'(x) < 0$ on $(-1, 2)$ and $(5, \infty)$
F. $f''(x) > 0$ on $(-\infty, -3)$
G. $f''(x) < 0$ on $(-3, 2)$ and $(2, \infty)$

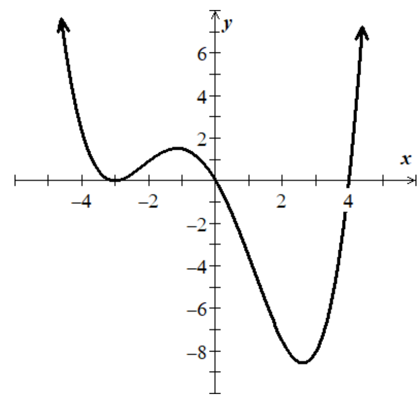


9. The graph of a function f is shown on the right. Fill in the chart with $+, -$, or 0.

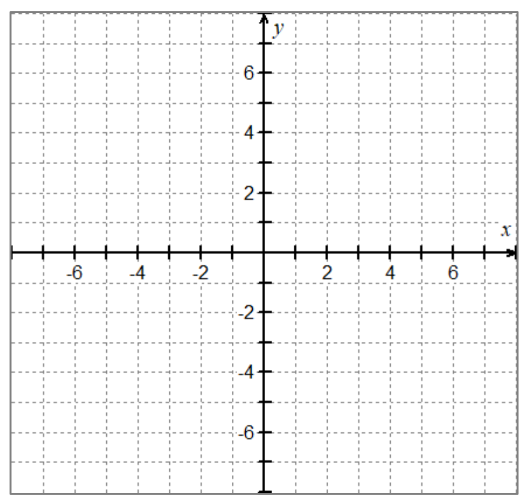
Point	$f(x)$	$f'(x)$	$f''(x)$
A			
B			
C			
D			



10. The graph of $f''(x)$, the second derivative of f is shown at right. Use the graph to determine The x-coordinate of the inflection points. Justify your answer.



11. Use the information from g, g', and g'' to sketch a graph of the function $g(x) = \frac{x}{x^2-9}$.



5.6 Solving Optimization Problems Homework

Problems 1 – 9, solve each question and justify your reasoning.

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1. A farmer has 350 feet of fencing to enclose 2 adjacent horse corrals. What dimensions should be used so that the enclosed area will be a maximum?

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2. A rectangle has its base on the x-axis and its two upper corners on the parabola $y = 12 - x^2$. What is the largest possible area of the rectangle?

---

3. The owner of an orange grove estimates that if 24 trees are planted per acre, then each mature tree will yield 600 oranges per year. For each additional tree planted per acre, the number of oranges produced by each tree decrease by 12 per year. How many trees should be planted per acre to obtain the most oranges per year?

4. A poster is being designed that is to contain 150 in.^{2} of printed material with margins of 2 inches at the top and bottom and 3 inches at each side. What overall dimensions will minimize the amount of paper used?

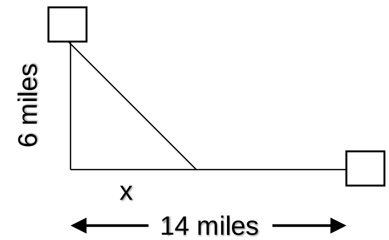
---

5. Find the dimensions of the rectangle with a maximum area inscribed in a semi-circle with a radius of 8 inches.

---

6. The size of a bacteria population found in some food grows at a rate of $P(t) = \frac{8000t}{80 + t^2}$ where t is measured in weeks. Determine when the bacteria will reach its maximum size. What is the maximum size of the population?

7. An offshore well is 6 miles off the coast. A gas line is to be laid from the well to a refining station that is 14 miles down the coast. The cost to lay the line is $23,000 per mile along the shoreline and $32,000 per mile under the ocean. How should the gas line be laid at the least expensive cost? What is the cost?



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8. A boat leaves a dock at 2:00 pm and travels due south at a speed of 20 kilometers/hour. Another boat has been heading due east at 15 kilometers/hour and reaches the same dock at 3:00 pm. At what time were the two boats closest together?

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9. On a given day, the flow rate F (cars per hour) on a congested roadway is given by $F(v) = \frac{v}{16+0.02v^2}$, where v is the speed of the traffic in miles per hour. What speed will maximize the flow rate on the road? Round your answer to the nearest mile per hour.