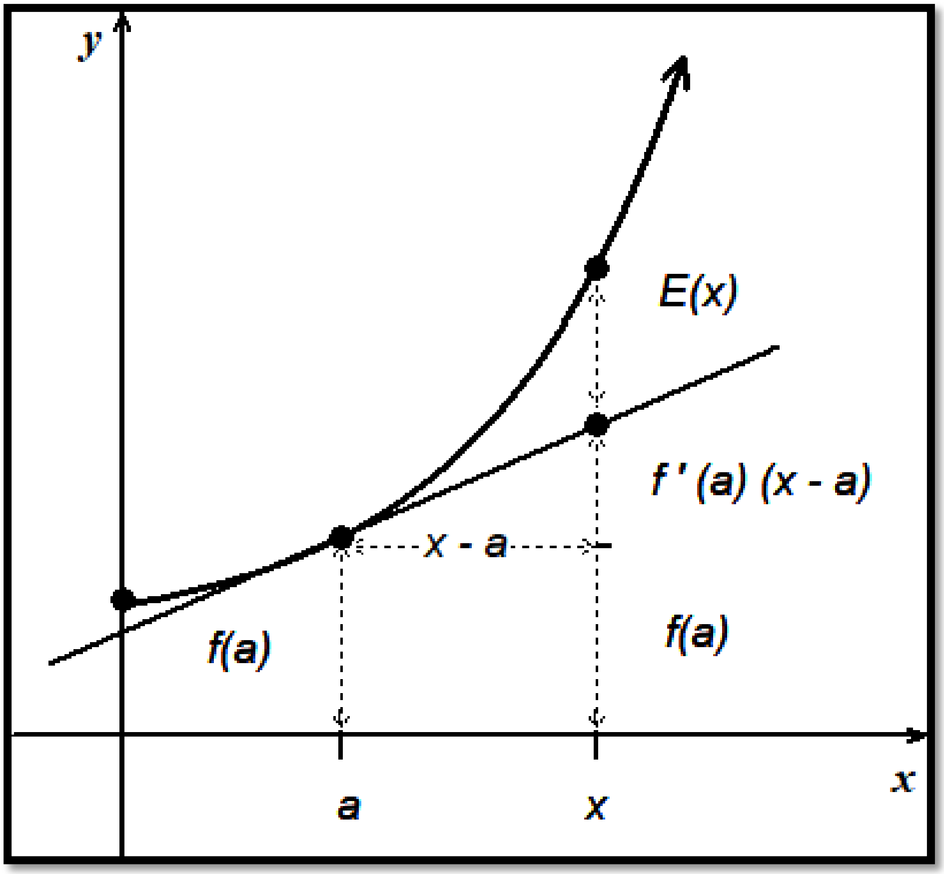


Name ____________________________________

UNIT 4:

CONTEXTUAL APPLICATIONS OF DIFFERENTIATION HOMEWORK

Flamingo Math

4.1 Interpretations of the Derivative in Context Homework

1. The temperature, T, in degrees Fahrenheit, of a pizza placed on a serving dish is given by $T = f(t)$, where t is the time in minutes since the pizza was taken out of the oven.

A. Is $f'(t)$ positive or negative? Justify with a reason.

B. What are the units of $f'(5)$?

C. What is the meaning of $f'(5) = -3$ in context to the scenario?

---

2. The cost, C (in dollars) to produce g gallons of fresh-squeezed orange juice can be expressed as $C = f(g)$. Using the proper units, explain the meaning of the following statements in terms of the orange juice.

A. $f(500) = 2150$

B. $f'(500) = 0.84$

---

3. You have an inheritance of \$100,000 from your grandparents, but you must invest the money in a mutual fund until you are 21 years old, that will pay an annual interest rate of $r\%$ compounded continuously. If you are 17 years old now, your balance, in dollars, can be defined as $B = A(r)$. Interpret the following statements related to your inheritance, use the appropriate units.

A. $A(7) \approx 13230$

B. $A'(7) \approx 3307.50$

4. The Apollo 11 Crew left Earth for the moon on July 16, 1969, at 9:32 a.m. in a Saturn V rocket. They experienced forces of about 1.2g at liftoff and reached a maximum force of about 3.9g. One g is the force per unit mass due to gravity at the Earth's surface, defined as 9.80665 m/s^2. Let $F = p(h)$ represent the amount of g-force pressure, p on the astronauts at a height h, in meters. Explain what each of the following quantities mean in relation to the astronauts. Give units for the quantities.

A. $p(500) = 3.2$

B. $p'(500) = 0.2$

C. $p'(h) = -1$

---

5. A law firm pays a third-party to pick up confidential documents and deliver them to a shredding service. This cost, C, in dollars, is a function of the weight of paper, w, in pounds, that is shredded for each client's case, such that $C = f(w)$. Describe the meaning of each quantity.

A. $f(65) = 45$

B. $f'(80) = 1.25$

C. $[f^{-1}(200)]' = 2.5$

---

6. When a patient is prescribed an antibiotic to fight off infection, the dose, D, in milligrams is a function of the patient's weight, in pounds. Let $D = f(w)$ define the dosage. Interpret the meaning of each statement.

A. $f(135) = 120$

B. $f'(135) = 2.5$

C. Estimate the dosage for a person weighing 140 pounds.

7. During a hurricane in Central Florida, the amount of rainfall, R, in centimeters, is a function of time. Let $R = f(t)$ define the amount of rain recorded by meteorologists beginning at midnight. Give an interpretation of the statements below, using the appropriate units.

A. $f(6) = 4.8$

B. $f'(6) = 0.5$

C. $f^{-1}(5) = 14$

D. $(f^{-1})'(5) = 1.5$

---

8. Nate took an initial dose of his prescription antibiotic medication. The amount of antibiotic, in milligrams, in Nate's bloodstream after t hours is given by $m(t) = 18e^{-0.75t}$. Find the instantaneous rate of change of the remaining amount of medication after 1 hour. Explain the meaning of your findings as it relates to Nate.

---

9. The population of Eulerville can be modeled by $P = f(t)$ where the population, P, is in thousands and t is the number of years since 2010. Explain the meaning of each statement in context to the city of Eulerville.

A. $f'(4) = 3$

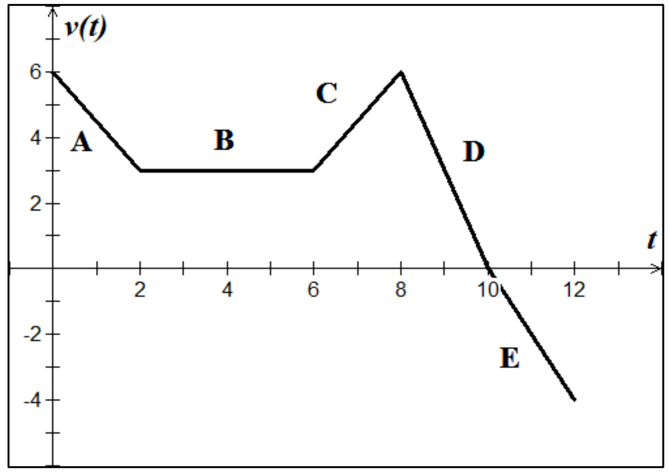
B. $f^{-1}(47.5) = 8$

C. $(f^{-1})'(47.5) = 0.3$

4.2 Straight Line Motion - Connecting Position, Velocity & Acceleration Homework

Problems 1 – 2, Use the velocity vs. time graph below to answer the questions.

1. In the velocity graph, at right, let the units for velocity be in feet per second.



	A	B	C	D	E
Acceleration (slope) (+, −, 0)					
Acceleration (yes/no)					
Velocity/Acceleration (+, −)					
Speed up, slow down, constant					

2. Use the graph of the velocity function above to find the following. Show the computations that lead to your answer, include units.

A. How far did the particle move in the first 10 seconds?

B. What is the average velocity over the 12 second time interval?

C When does the particle change directions?

Problems 3 – 8, Evaluate each scenario, in context.

---

3. If a calculus book is dropped from a height of 100 feet, its height s at time t is given by the position function $s(t) = -16t^2 + 100$, where s is measured in feet and t is measured in seconds. Find the average velocity over the time interval [1, 2].

---

4. Hayden is standing on a ledge that is 32 feet above a mountain stream. At time $t = 0$, he jumps from the ledge into the water below. Hayden's position at any time $t \geq 0$ can be modeled by the function $s(t) = -16t^2 + 16t + 32$, where s is measured in feet, and t is measured in seconds.

A. When does Hayden hit the water?

B. What is Hayden's velocity at impact?

---

5. A small rocket is shot upward from the surface of the earth with an initial velocity of 120 meters per second. The position function of the rocket at $t \geq 0$ is modeled by the function $s(t) = -4.9t^2 + 120\,t$.

A. When is the velocity zero?

B. When does the rocket hit the ground?

C. What is the velocity of the rocket at impact?

D. At $t = 15$ seconds, is the rocket speeding up or slowing down? Justify.

6. A drone delivery service drops a package when the drone is 120 ft above the ground, rising at 12 ft/sec. The position function is $s(t) = -16t^2 + 12t + 120$.

A. How long will it take for the package to hit the ground?

B. What is the speed of the package at impact?

---

7. A potato is fired vertically upward from a potato cannon with an initial velocity of 84 ft/sec from a building that is 224-feet high. The position function is given by $s(t) = -16t^2 + 84t + 224$.

A. How long will it take for the potato to reach its maximum height?

B. What is the maximum height?

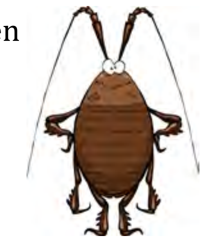
C. How long will it take the potato to reach its starting height on the way down?

D. What is the velocity when the potato passes the starting point on the way down?

---

8. A particle is moving along a line so that at any time t, where $0 \leq t \leq \pi$, the position of the particle can be modeled by $s(t) = 5 - 0.5t^2 - 3\cos(t)$. When acceleration is zero, what is the velocity of the object?

9. Randy the Roach is back. He is moving along a horizontal line (it could be your kitchen counter at 2 a. m.) in a smooth and continuous manner. His velocity, v, in inches per minute, is a differentiable function and has been tracked by your home security system. Randy's velocity is shown for $0 \leq t \leq 10$ minutes in the table below.



t	0	0.5	1	2	2.5	4	5	6.5	8	10
$v(t)$	-1.7	0.5	0.5	.75	-0.5	-0.25	0.5	1.25	1.75	0.25

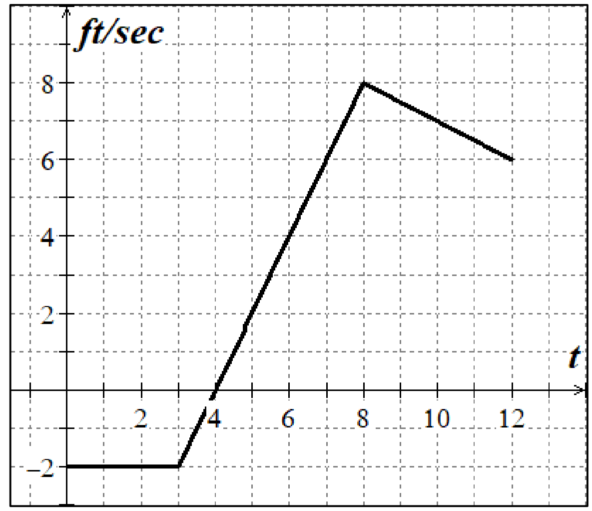
A. At what time(s) is Randy moving to the left? Explain how you know.

B. Is there any time(s) during the interval $0 \leq t \leq 10$ minutes when Randy is at rest? Explain your answer.

C. Use the data from the table to find an approximation for $v'(7)$. Explain the meaning of $v'(7)$ in terms of Randy's movement. Show the computations that lead to your answer, and indicate the units of measure.

D. Find Randy's average acceleration for $4 \leq t \leq 5$ minutes. Explain what this means in terms of the Randy's velocity on the interval.

10. The graph at right shows the velocity $v(t)$ of a particle, in ft/sec, moving along a horizontal line for time, $0 \le t \le 12$ seconds.



A. On what open intervals or at what time $0 < t < 12$ is the particle at rest? Justify.

B. On what open intervals $0 < t < 12$ is the particle moving to the right? Justify.

C. On what open intervals or at what time(s) $0 < t < 12$ is the particle's speed increasing?

D. Does the particle slow down? If so, when? Explain how you know.

E. How far does the particle move during the first four seconds, and in which direction? Show the computations that lead to your answer.

F. What is the average velocity over the time $4 < t < 12$ seconds? Show the computations that lead to your answer.

G. What is the particle's position at the end of the 12 seconds? Show your computations that lead to the answer.

4.3 Rates of Change in Other Applications Homework

Problems 1 – 7, evaluate the marginal analysis scenarios.

---

1. ***Nole Quarters*** is located just blocks away from Doak Campbell Stadium on the FSU Campus, in Tallahassee, FL. There are 65 student rental apartments available. If x apartments are rented then the monthly profit, in dollars, is modeled by the profit function, $\pi(x) = -7x^2 + 840x - 12{,}500$.

Photo by: Sabrina Adams

A. How many apartments should be rented to maximize profits?

B. Evaluate profit for $\pi(0)$ and $\pi(65)$

C. What is the maximum monthly profit earned?

---

2. You are listening to a lecture on the Battle of Gettysburg in your APUSH class; and, are reminded of the laws of retention from psychology. You love math, so you tune out and begin to calculate the percentage of information you will forget, t minutes after the lecture began, using the formula for retention, given by $R(t) = \frac{44}{1 + e^{0.02t}}$. A maximum occurs when $t = 0$ and decreases as time passes. Hey, are you paying attention to the lesson?

A. Find how the retention rate is changing 15 minutes after the lesson begins.

B. Explain the meaning of your answer in context to the problem.

---

3. A paper manufacturer has 150 employees at a paper mill that can produce $Q(x)$ units of paper, for $100 \leq x \leq 200$ employees at the mill. Their production output is modeled by the function $Q(x) = 18000 + 2x^2$ units. What is the instantaneous rate of change of paper production in units per employee?

4. Let $C(x)$ represent the cost, $R(x)$ the revenue, and $\pi(x)$ the total profit, in dollars of producing x units of a widget. If $C'(400) = 18$ and $R'(400) = 15$, what is the approximate profit or loss by the 401^{st} widget? Show your computations that lead to your answer.

---

5. The revenue from selling x items is $R(x) = 650x - x^2$, the total cost is $C(x) = 175 + 12x$. Write a function $\pi(x)$ that gives the total profit earned. At what quantity will the profit be maximized?

---

6. A large video game developer has fixed costs of \$2.5 million, variable costs are \$0.05 million per game developed.

A. Write a function for total cost, $C(x)$, as a function of x games created .

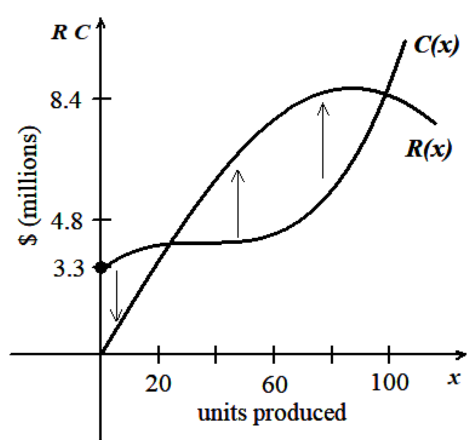
B. The games in (A) are sold for \$0.175 million each. What is the revenue formula as a function of x ?

C. Write the formula for the profit function $\pi(x)$.

D. What does $\pi'(x)$ mean in context to the game developer's profits?

---

7. The figure, at right, shows the revenue and costs, for the production of a commodity.



A. What are the fixed costs?

B. At what quantity is the profit maximized? Explain.

C. What is the maximum profit earned? Show work.

Problems 8 – 14, find the rates of change that are non-motion related.

---

8. A navigational buoy is anchored in the intercoastal waterway to mark the channel for boat traffic. When a jet ski passes by, the buoy oscillates up and down rapidly and can be modeled by $s(t) = 12 + 0.25\sin(\pi x)$ for time $0 \le t \le 8$ seconds and $s(t)$ is in inches.

A. Find the velocity $s'(t)$ of the buoy.

B. When does the buoy reach its maximum height for the first time during $0 < t < 8$? Find the height.

C. What is the $v(3)$? Is the buoy moving up or down? Explain your reasoning.

---

9. The cost, in dollars, of processing x pounds of sugarcane can be modeled by the cost function $C(x) = 0.001x^3 - 0.12x^2 + 6x + 230$ **.**

A. What is the meaning of the instantaneous rate of change of the cost when 100 pounds of sugarcane are processed?

B. What is the average cost of processing 100 pounds of sugarcane?

---

10. The population P, of *Eulerville*, in hundreds of residents, was approximately $P(t) = 632e^{0.001t}$ for December 31, 2018. At what rate was the population of *Eulerville* growing at the end of 2019?

---

11. Celeste takes a daily diabetes drug to maintain the levels of sugar in her bloodstream. The amount of medication, in milligrams, present in her bloodstream after t hours is measured by $m(t) = 22e^{-0.84t}$. Find the rate of change of the remaining medication after 3 hours?

12. The number of automobile thefts $B(t)$, reported each day in a certain parking lot increases for time between midnight and 6 a.m. The number of break-ins can be modeled by $B(t) = 4\sin\left(\frac{\pi x - 82}{15}\right) + 3$.

A. At what time are the number of break-ins at the greatest?

B. How many are reported at this time?

---

13. A re-chargeable battery pack is manufactured and sells for $45 per battery. The production and manufacturing process has marginal costs as shown in the table below. At how many units, *x*, does the profit appear to be at a maximum? In what interval(s) do these quantities appear to lie? Explain how you know.

x	0	10	20	30	40	50	60	70
MC ($/unit)	48	35	31	16	18	24	46	52

---

14. Use the following table of values for the number of gallons, $V(t)$, of oil that remain in a 55-gallon oil barrel, t hours after a forklift operator dropped the container onto the loading dock and pierced a hole in the container.

t	0	1	2	3	3.5	5	6	8
$V(t)$	55	54.6	52.8	49.6	42.1	40.8	38.4	37.9

A. Estimate $V'(2.5)$. Explain the meaning in context to the problem.

C. For what interval of time is the oil leaking the fastest? Estimate the instantaneous rate of change, for this time.

4.4 Related Rates Homework

1. Assume x and y are both differentiable functions of t. Find $\frac{dx}{dt}$ given $x = -1$ and $\frac{dy}{dt} = 8$. For the relation: $4x^2 + 3y^3 = 28$.

---

2. Water is falling on a surface, wetting a circular area that is expanding at a rate of $10\ mm^2/\text{sec}$. How fast is the radius of the wetted area expanding when the radius is 116 mm? (Round approximations to four decimal places.)

---

3. A spherical balloon is being inflated. The radius of the balloon is increasing at a rate of $\frac{1}{\pi}$ inches per second. Find the rate at which the volume is increasing when the radius is 1 inch.

---

4. All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is 10 centimeters?

5. An aircraft is climbing at a $30°$ angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 miles per hour?

---

6. A man 6-ft tall walks at a rate of 5 ft/sec. away from a lamppost that is 22-ft high. At what rate is the length of his shadow changing when he is 60 feet away from the lamppost?

---

7. The height of a cylinder with a radius of 4 cm is increasing at a rate of 2 centimeters per minute. Find the rate of change of the volume with respect to time when the height is 10 centimeters.

8. A conical water tank is 24 feet high and has a radius of 10 feet at the top. If water flows into the tank at the rate of 20 cubic feet per minute, how fast is the depth of the water increasing when the water is 16 feet deep?

---

9. Eleni and Analisa are driving to Ms. A's wedding. Eleni is going east at a rate of 50 mph and Analisa is driving south at 38 mph. At what rate is the distance between the cars changing when Eleni is 1 mile and Analisa is ¾ mile from the wedding?

---

10. A hot air balloon rises at a rate of 8 feet per second from a point on the ground 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet above the ground.

11. A trough is 10 feet long and the ends have the shape of isosceles triangles that measure 3 feet across at the top and have a height of 1 foot. If the trough is being filled with water at a rate of 12 ft^3 per minute, how fast is the water level rising when the water is 6-inches deep?

---

12. A container is the shape of an inverted right circular cone has a radius of 5 inches at the top and a height of 7 inches. At the instant when the water in the container is 3 inches deep, the surface level is falling at the rate of – 7 in/sec. Find the rate at which water is being drained.

---

13. Sand is falling from a conveyor at a rate of 28 ft^3/min onto the top of a conical pile. The height of the pile is always ½ of the base diameter. How fast is the height changing when the pile is 12 feet high?

4.5 Linear Approximation Homework

Problems 1 – 4, Find the tangent line approximation of each function at the given value $x = a$.

1. $f(x) = \tan x$; at $a = \frac{\pi}{4}$	**2.** $y = e^{\sqrt{x}}$; at $a = 1$
3. $g(x) = \frac{1}{\sqrt{1+x}}$; at $a = 3$	**4.** $y = \tan^{-1} x$; $a = 1$

5. Let f be a differentiable function such that $f(-2) = 3$ and $f'(-2) = 6$. The tangent line to the graph of f at $x = -2$ can be used to find an approximation to a zero of the function, f. Find the approximation for the value of x such that $f(x) = 0$.

6. Find the linear approximation for $f(\theta) = \sin^2 \theta$ for $\theta = \frac{\pi}{4}$. Use the linear approximation to estimate $\theta = 0.8$ and calculate the error. Determine whether the approximation overestimates or underestimates the function.

7. Find the linearization of the function $f(x) = \sqrt{x+3}$ at $x = 1$ and use it to approximate $\sqrt{4.05}$. Determine whether the approximation is an overestimate or underestimate. Explain.

---

8. Let f be a function that is twice differentiable for all real numbers. The table below gives values of f for several points in the interval $1 \leq x \leq 10$.

$\boldsymbol{x}$	1	3	6	9	10
$\boldsymbol{f(x)}$	2	4	-3	1	3

A. Estimate $f'(5)$. Show work that leads to your answer.

B. Suppose $f'(6) = 2$ and the function f is concave up on the closed interval $6 \leq x \leq 9$. Use the tangent line at $x = 6$ as an approximation to show that $f(8) \leq 1$.

C. Write the secant line for the graph of f on $6 \leq x \leq 9$ to show that $f(8) \geq -\frac{1}{3}$.

9. Use the table, at right, to complete the following.
Given $P(x) = f\big(g(x)\big)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	-2	1	4
-1	1	-1	-2	1
0	-1	2	-1	-2

A., Write the equation of the tangent line at $x = -1$.

B. Use the method of linear approximation to estimate the value of $P(-1.5)$.

C. If $P(-2) = 5$, what does this value tell you about the behavior of the graph of $P(x)$ when $x = -1.5$? Explain how you know.

10. Use linearization to estimate the given numbers. Find the error of the approximation.

A. $\ln(1.06)$

B. $(8.05)^{2/3}$

11. The local linear approximation to the function g at $x = -3$ is $y = 4x - 7$.
What is the value of $g(-3) + g'(-3)$?

12. Find a linear approximation for $f(x) = 2x\,e^{2x-6}$ at $x = 3$. Then, use your equation to estimate $f(3.1)$. Find the error.

---

13. Let $g(x) = x^2 - x - 12$. The tangent line to the graph of g at $x = 2$ is used to approximate values of $g(x)$. Which of the following is the greatest value of x for which the error from this tangent line approximation is less than 0.5 ?

A. 1.3 **B.** 2.6 **C.** 2.7 **D.** 2.8

---

14. Given the implicit curve, $x^2 + y^3 = 2x^2 y$. At the point $(1, 1)$ write the equation for the tangent line and use it to approximate $y(1.2)$.

---

15. Given $f'(x) = \frac{6x^4}{x^2+1}$ and $f(1) = 8$. Use a linear approximation to estimate $f(1.02)$

4.6 L'Hospital's Rule Homework

Use L'Hospital's Rule to evaluate the limit, if it exists, or state that L'Hospital's Rule does not apply.

1. $\lim_{x \to -3} \frac{x^2 - 9}{6 - x - x^2}$

2. $\lim_{x \to 2} \frac{3x^2 - 5x - 2}{x - 3}$

3. $\lim_{x \to 1} \frac{\sqrt{x + 3} - 2}{x^3 - 4x + 3}$

4. $\lim_{x \to 0} \frac{\sin 5x}{2x}$

5. $\lim_{x \to \infty} \frac{5x - x^2}{3x^2 + 2x - 4}$

6. $\lim_{x \to -\infty} \frac{x}{e^x}$

7. $\lim_{x \to 0} \frac{\sin 3x}{\sin 5x}$

8. $\lim_{x \to 0} \frac{x}{1 - e^x}$

9. $\lim_{x \to 0} \dfrac{x^2}{1 - \cos x}$

10. $\lim_{x \to \infty} \dfrac{e^{2x} - x - 1}{x^2}$

11. $\lim_{x \to 0} \dfrac{\tan x}{x}$

12. $\lim_{x \to 0} \dfrac{e^x - 2}{\sin x}$

13. $\lim_{x \to \infty} \dfrac{x^{2/3} + 2x}{x^{5/3} - x}$

14. $\lim_{x \to 0} \dfrac{x^3}{\sin x - x}$

15. $\lim\limits_{x \to 1} \dfrac{\tan^{-1} x - \frac{\pi}{4}}{x - 1}$

16. $\lim\limits_{x \to 1} \dfrac{\ln x^2}{x^2 - 1}$

17. $\lim\limits_{x \to \infty} \dfrac{x}{\sqrt{x^2 + 1}}$

18. $\lim\limits_{x \to \infty} \dfrac{\ln x}{\sqrt{x}}$

Questions 19 – 22, Multiple Choice Practice

19. Find $\lim\limits_{x \to 1^+} \left(\dfrac{x}{\ln x} \right)$

A. 0 **B.** 1

C. e **D.** $+\infty$

20. What is $\lim_{x \to 0} \frac{e^x - 1}{\tan x}$

A. -1 **B.** 0

C. 1 **D.** The limit does not exist

---

21. If $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} g(x) = +\infty$, and $f'(x) = 1$ and $g'(x) = e^x$, what is $\lim_{x \to +\infty} \frac{f(x)}{g(x)}$?

A. 0 **B.** 1

C. e **D.** The limit does not exist

---

22. Evaluate.

$$\lim_{x \to 1} \left(\frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}} \right)$$

A. 2 **B.** 1

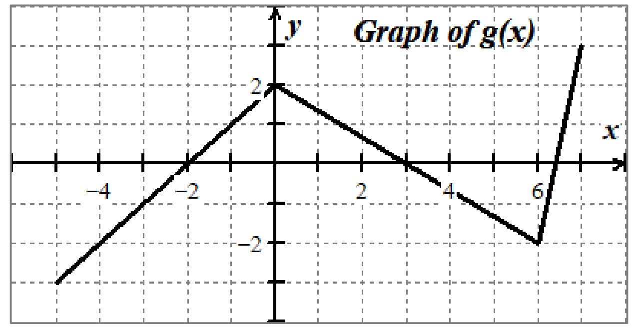
C. $\frac{1}{2}$ **D.** The limit does not exist

For each of the following, find the limit, if it exists, and justify your reasoning.

---

23. Use the graph of $g(x)$, shown at right to find the limit, if it exists

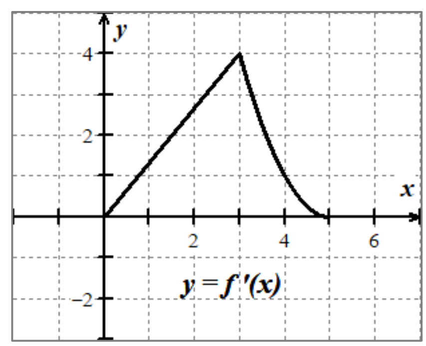
$$\lim_{x\to -2} \frac{g(x+5)}{x^2+3x+2}$$



---

24. The function $f(x)$ is differentiable and the graph of $f'(x)$ is shown at right. Given that $f(3) = 0$, find the following. Justify your reasoning.

$$\lim_{x\to 3} \frac{x^2 - 9}{f(x)}$$



---

25. The function $h(x)$ is twice differentiable and $h(-1) = 0$. The differential equation for the curve $y = h(x)$ is given by $\frac{dy}{dx} = 2^x(y - 3)$ Find the limit, if it exists. Explain your reasoning.

$$\lim_{x\to -1} \frac{x + e^{x+1}}{x^2 + x}$$