

Name: _____

UNIT 3:

DIFFERENTIATION

Composite, Implicit, & Inverse Functions

Part 2

HOMEWORK

3.1

The Chain Rule
Homework

Name _____

Date _____ Period _____

Problems 1 - 11, find the derivative of the function.

1. $y = (3x - 2)^4$

2. $f(x) = 5(2 - x^2)^3$

3. $y = (6 + 3x)^{2/3}$

4. $f(x) = \sqrt{x + 3}$

5. $g(x) = \sqrt[3]{x^2 - 9}$

6. $y = \frac{-2}{(x^2 - 3x - 4)^2}$

7. $f(x) = \left(\frac{3}{3-x}\right)^2$

8. $y = \sqrt[3]{2x^2 - 5x + 3}$

9. $y = \frac{1}{2}x^2\sqrt{4-x^2}$

10. $f(x) = \frac{x}{\sqrt{x^2+1}}$

11. $s(t) = \left(\frac{t-3}{t^2+2}\right)^2$

Problems 12 - 17, find the derivative of each trigonometric function using the Chain Rule.

12. $y = \sin e^{2x}$

13. $y = \tan(\ln x)$

14. $f(x) = \csc(\theta^2)$

15. $y = \cos \sqrt{x}$

16. $g(\theta) = 4 \sin^2(\pi\theta)$

17. $y = \sin(\cos 3x^2)$

Problems 18 - 21, find the equation of the tangent line at the given point.

18. $y = \sqrt{x^2 + 3x + 6}$ at $(2, 4)$

19. $y = (x^2 + x - 2)^2$ at $(-1, 4)$

20. $y = \sin(2x)$ at $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

21. $y = (9 - x^2)^{1/3}$ at $(1, 2)$

Problems 22 - 31, find the derivative.

22. $h(x) = e^x \sin^2 x$

23. $f(x) = \ln(x^3) \cos x$

24. $g(x) = \ln(x^3 - 2x)$

25. $y = \sin(x^2 + 3x)$

26. $y = x \cos(\ln x)$

27. $f(x) = \sqrt{\sin x \cos x}$

28. $g(x) = \frac{\pi}{x^3} - \cos x \tan x$

29. $h(x) = \cos\left(\frac{1}{x}\right)$

30. $y = \tan(\theta^2 - 3\theta)$

31. $f(x) = (x^2 + 3e^x)^{\frac{3}{2}}$

3.2

Differentiation from Tables, Graphs
and Symbolically Homework

Name _____

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Let $f(x)$, $f'(x)$, $g(x)$, $g'(x)$ be differentiable functions whose values are given in the table below. Find the symbolic differentiation. Then use the values from the table to find the derivative at the designated point.

x	-1	0	1	2
$f(x)$	-1	3	-2	0
$f'(x)$	-3	4	1	5
$g(x)$	2	1	5	-2
$g'(x)$	7	-3	0	-1

1. $\frac{d}{dx}(f + g)$ Evaluate when $x = 1$

2. $\frac{d}{dx}(f - g)^2$ Evaluate when $x = 0$

3. $\frac{d}{dx}\left(\frac{1}{f-g}\right)$ Evaluate when $x = 2$

4. $\frac{d}{dx}\sqrt{f + g}$ Evaluate when $x = 1$

5. $\frac{d}{dx}(f \cdot g)$ Evaluate when $x = 0$

6. $\frac{d}{dx}(f^2)$ Evaluate when $x = 2$

7. $\frac{d}{dx}(fg)^{-1}$ Evaluate when $x = -1$

8. $\frac{d}{dx}(fg)^{\frac{1}{2}}$ Evaluate when $x = 0$

9. $\frac{d}{dx}\left(\frac{g}{f}\right)$ Evaluate when $x = 1$

10. $\frac{d}{dx}(f(g(x)))$ Evaluate when $x = -1$

11. $\frac{d}{dx}(f - g)$ Evaluate when $x = -1$

12. $\frac{d}{dx}(gf)$ Evaluate when $x = 1$

13. $\frac{d}{dx}(fg)^2$ Evaluate when $x = 0$

14. $\frac{d}{dx}(f \cdot g)^3$ Evaluate when $x = -1$

15. $\frac{d}{dx}\left(\frac{f}{\sqrt[3]{g}}\right)$ Evaluate when $x = 0$

16. $\frac{d}{dx}\left(\frac{1}{f^2}\right)$ Evaluate when $x = 0$

Problems 17 – 20, use the table of values to calculate the derivative of the function at the given point.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	9	-2
3	0	-1	2	3
6	-1	1/2	3	4

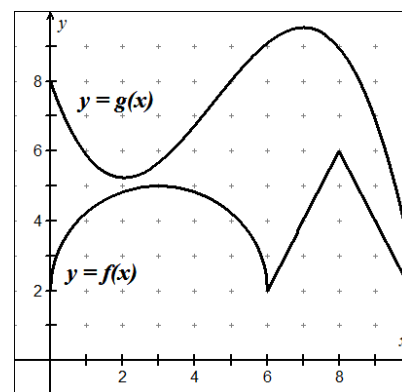
17. $H(x) = f(g(x))$. Find $H'(6)$

18. $K(x) = g(\sqrt{x})$. Find $K'(9)$

19. $J(x) = e^{f(x)}$. Find $J'(3)$

20. $R(x) = \ln(x^2 \cdot \sqrt{g(x)})$. Find $R'(1)$

Problems 21- 22, let $H(x) = f(g(x))$, where the graphs of f and g are shown at right. Estimate each derivative. Justify your analysis algebraically.



21. $g'(f(6))$

22. $H'(6)$

3.3

Implicit Differentiation
Homework

Name _____

Date _____ Period _____

Problems 1 - 8, find dy/dx by implicit differentiation.

1. $8x^2 + y^2 = 10$

2. $\sqrt{x} + \sqrt{y} = 100$

3. $\frac{x}{y} + \frac{y}{x} = 3y$

4. $5x^2 - xy - 4y^2 = 0$

5. $x e^y = 2xy + y^3$

6. $\sin x + 2 \cos 2y = 1$

7. $\cos(xy) = xy$

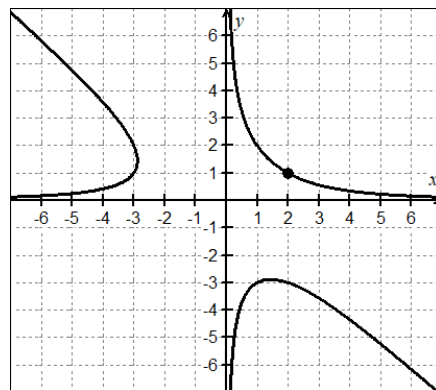
8. $2xy + \pi \cos y = 2\pi$

Problems 9 - 11, find the equation of the tangent line and the equation of the normal line to the curve at the indicate point.

9. $x^3 + 2xy - y^2 = 11$ at $(2,3)$

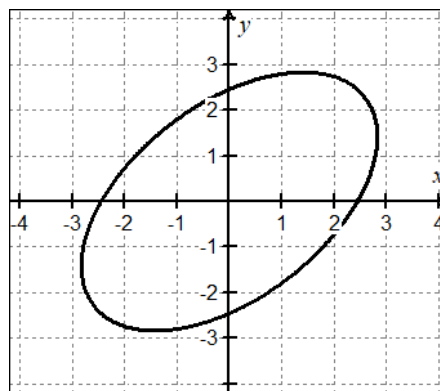
10. $\sqrt{xy} = 12y - x$ at $(9,1)$

11. $x^2y + y^2x = 6$ at $(2,1)$

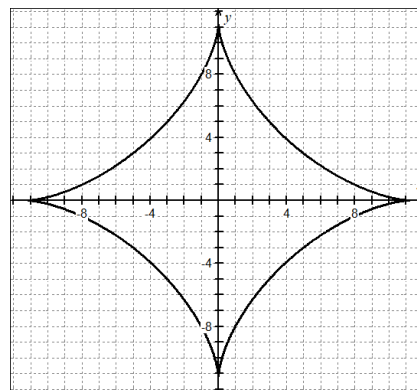


Problems 12 - 15, complete each of the following.

- 12.** Given $x^2 - xy + y^2 = 6$, find the x-intercepts. Then, show that the tangent lines to the curve at these intercepts are parallel.

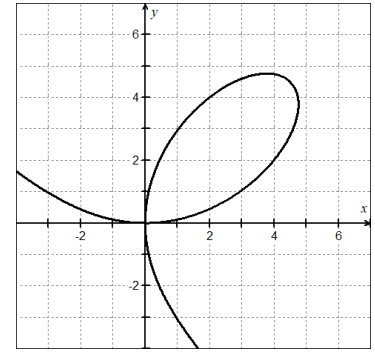


- 13.** Write the equations of the tangent line for $x^{2/3} + y^{2/3} = 5$ at $(8,1)$ and $(-8, 1)$. Are the tangent lines to the two points perpendicular? Explain.



14. Given $x^3 + y^3 - 9xy = 0$ find following.

A. The equation of the tangent lines at $(4, 2)$ and $(2, 4)$.



B. At what point does the curve have a vertical tangent line?

15. Find $\frac{d^2y}{dx^2}$ in terms of x and y , given $5x^2 - 2y^2 = 4$

3.4

Inverse Functions
Homework

Name _____

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Problems 1 - 4, find $(f^{-1})'(b)$ for each function.

1. $f(x) = x^3 - 1, b = 7$

2. $f(x) = x^3 + 2x - 1, b = 2$

3. $f(x) = \sin x, b = \frac{1}{2}$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$

4. $f(x) = \cos 2x, b = 1$ on $[0, \frac{\pi}{2}]$

Problems 5 - 6, Multiple Choice.5. If $f(x) = \frac{4}{x} + 2$ and g is the inverse of f , then $g'(10) =$

- A.
- -16
- B.
- $-\frac{1}{2}$
- C.
- $-\frac{1}{16}$
- D.
- $\frac{1}{2}$

6. If $(3,5)$ is a point on the graph of $y = f(x)$, a one-to-one function, and $f'(3) = 2$, then which of the following must be true?

- A.
- $\frac{d}{dx}[f^{-1}(3)] = \frac{1}{2}$
- B.
- $\frac{d}{dx}[f^{-1}(3)] = -\frac{1}{2}$
-
- C.
- $\frac{d}{dx}[f^{-1}(5)] = \frac{1}{2}$
- D.
- $\frac{d}{dx}[f^{-1}(3)] = -2$

Problems 7 – 10, Short Answer.

7. If $f(3) = 8$ and $f'(3) = 5$ what do you know about f^{-1} ?

8. Let $f(x) = x^3 + x - 2$, and let g be the inverse function. Evaluate $g'(0)$.

9. A function f and its derivative take on the values shown in the table. If g is the inverse of f , find $g'(6)$.

x	$f(x)$	$f'(x)$
2	6	$\frac{1}{3}$
6	8	$\frac{3}{2}$

10. Your parents bought you a brand new car at graduation! Well, we can dream, right? Assume that the value of the car is given by the function, $V(t) = 25,000e^{-0.1t}$, where v is the value, in dollars, of the car and t is the time, in years, from the date of purchase.

A. How much was the car worth at the dealership ($t = 0$)?

B. How much did the car depreciate during its fourth year (from $t = 3$ to $t = 4$)?

C. When you graduate from college, assuming you are on the 4-year plan, what is the instantaneous rate of change (in dollars per year) of the value at $t = 4$? Why is this different than your answer in part B?

D. When will the value of the car reach \$12,500?

Problems 11- 26 , Find the derivative of the function. Remember to use logarithmic properties before differentiating.

11. $f(x) = 5^x$

12. $y = 4^{3x}$

13. $y = 6^{2x-3}$

14. $y = x (7^x)$

15. $y = x(3^{-2x})$

16. $f(x) = x^2 3^x$

17. $f(t) = \frac{5^{2t}}{t}$

18. $f(\theta) = 2^\theta \cos(\pi\theta)$

19. $h(x) = 4^{-x/2} \sin(2x)$

20. $y = \log_2(3x + 1)$

21. $f(x) = \log_3(x^2 - 4x)$

22. $g(x) = \log_4(x^2 + 1)^3$

23. $y = \log_3 \sqrt{x^2 + 1}$

24. $y = \log_2 \sqrt[3]{2x - 4}$

25. $f(x) = \log_2 \frac{x^2}{x+1}$

26. $g(x) = \log \frac{x^2-1}{x}$

27. If $g(x) = 5x^9 - x^2 + 2x - 3$, find $(g^{-1})'(7)$ accurate to three decimal places.



3.5

Derivatives of Inverse
Trigonometric Functions Homework

Name _____

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Problems 1 - 12, differentiate.

1. $y = \tan^{-1}(e^x)$

2. $y = \cos^{-1} \sqrt{x}$

3. $h(x) = \arccos\left(\frac{x}{2}\right)$

4. $y = \cos^{-1}(x^3)$

5. $y = \arctan\left(\frac{1}{x}\right)$

6. $g(x) = \sin^{-1} x^2$

7. $h(x) = 2x \sin^{-1} x$

8. $y = \tan^{-1}(\cos x)$

9. $g(x) = x^2 \tan^{-1} 3x$

10. $f(x) = e^{\cos^{-1} x}$

11. $k(x) = 3 \arctan(e^{3x})$

12. $h(x) = \ln|\sin^{-1} x|$

13. Using a calculator, find an equation for the tangent line to the graph of $y = \sin^{-1}\left(\frac{x}{4}\right)$ at $x = 3$. Round your answer to three decimal places.

