DIFFERENTIATION:
Definition and Basic Derivative Rules

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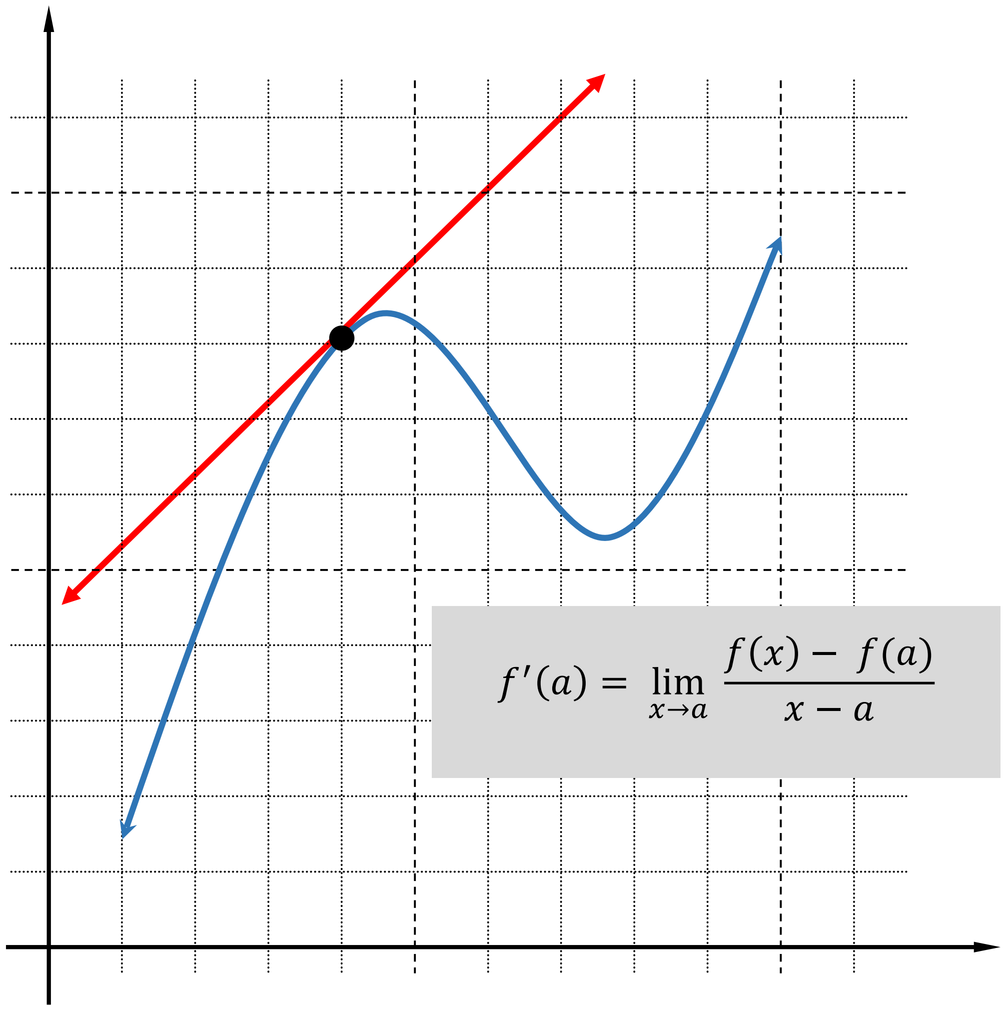
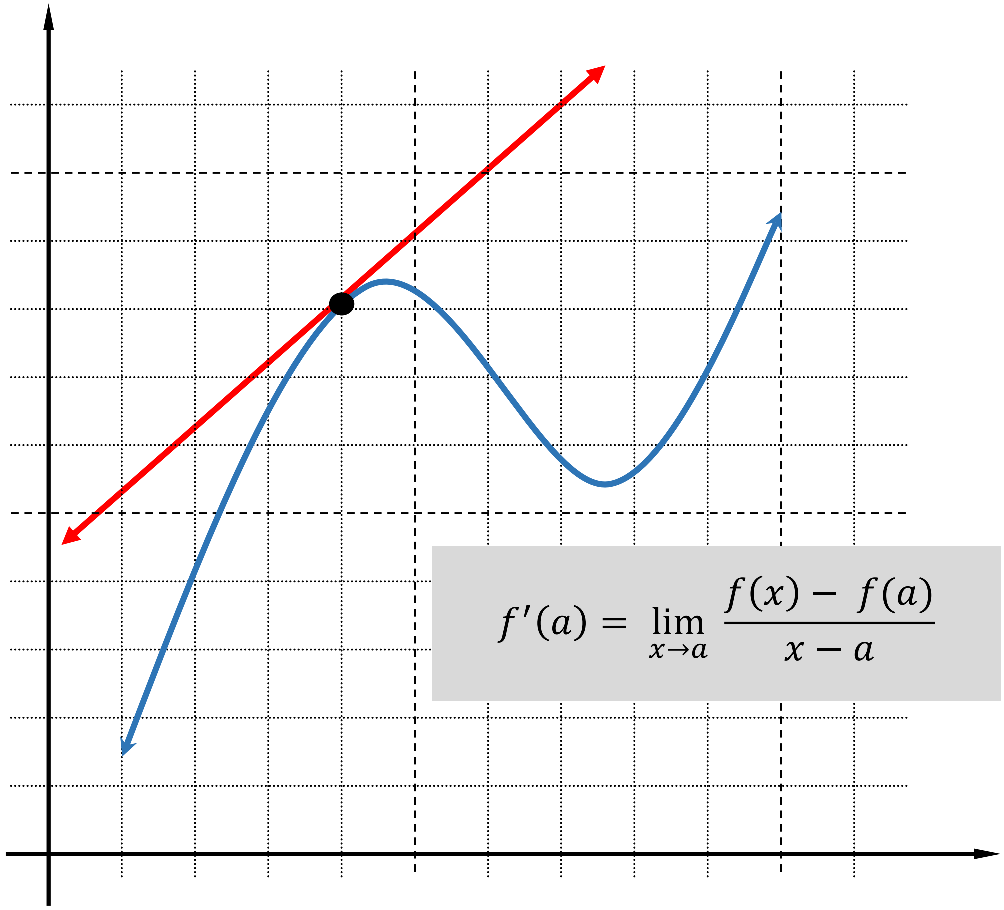


UNIT 2:

DIFFERENTIATION

Definition and Basic Derivative Rules



$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

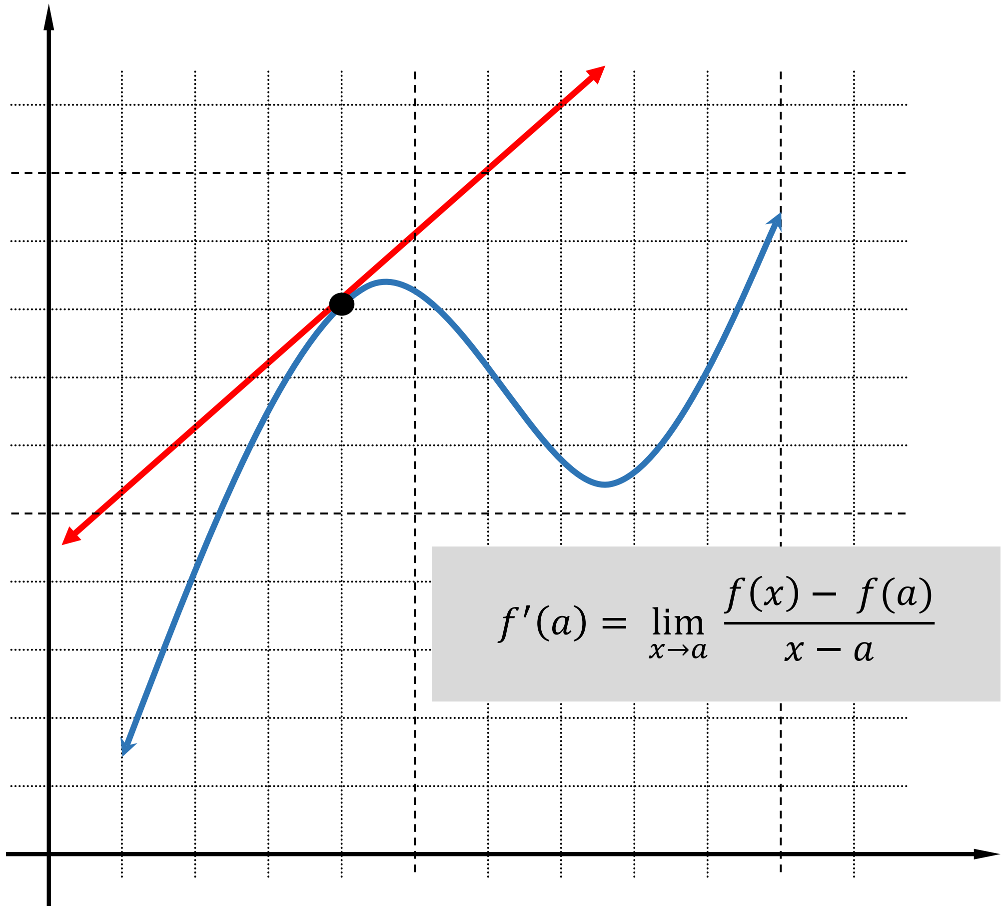
Name: ____

UNIT 2:

DIFFERENTIATION:

Definition and Basic Derivative Rules

NOTES



$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

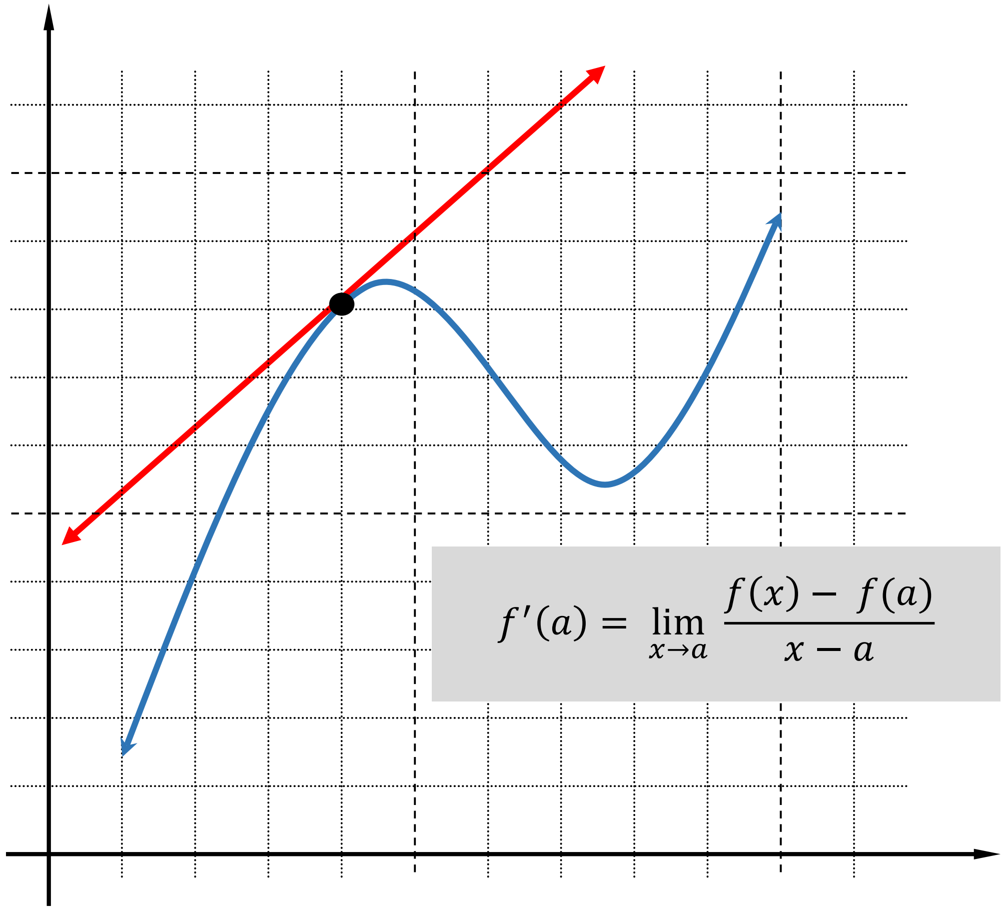
Name: ____

UNIT 2:

DIFFERENTIATION:

Definition and Basic Derivative Rules

HOMEWORK



UNIT 2:

DIFFERENTIATION:

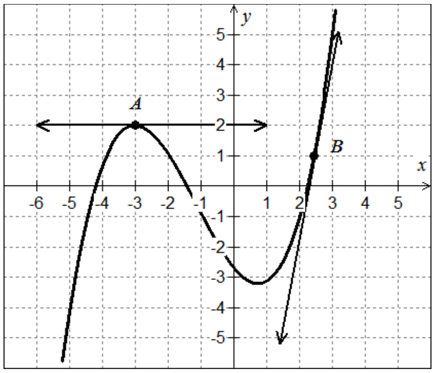
Definition and Basic Derivative Rules

ASSESSMENTS

2.1 Rates of Change and the Tangent Line Problem Homework

Problems 1–2, estimate the slope of the graph at the points *A* and *B*.

1.



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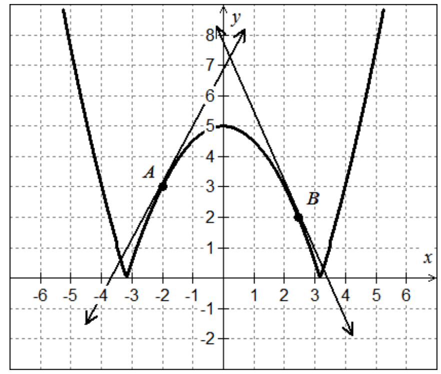
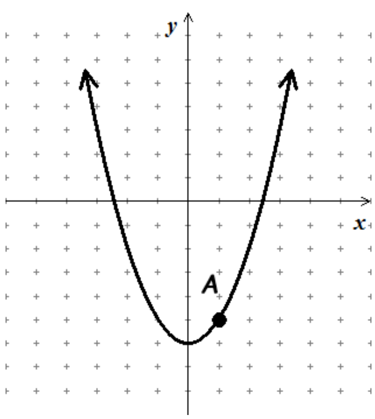
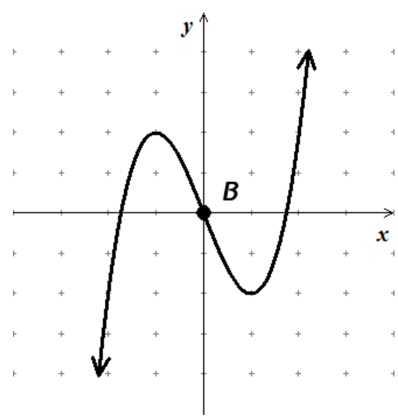


Problems 3–6, sketch tangent lines to the given functions at each of the labeled points. Use a straightedge, be precise. Extend each tangent line well beyond the point of tangency in each direction. Estimate the slope of the tangent line. Show your analysis algebraically.

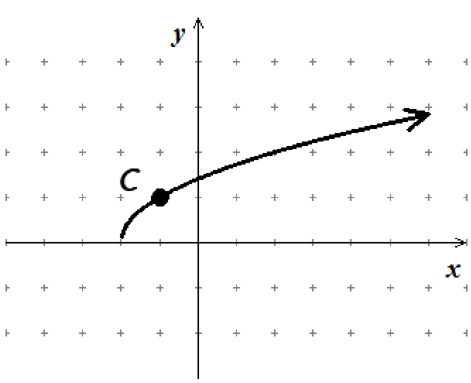
3. $f(x) = x^2 - 6$ at $(1, -5)$



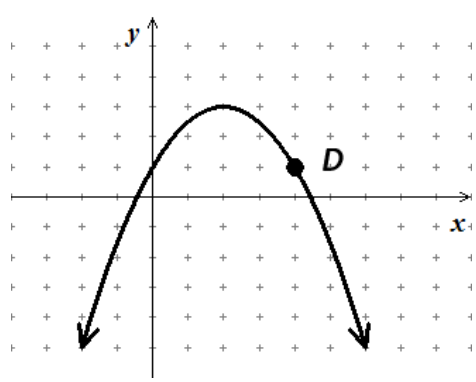
4. $f(x) = x^3 - 3x$ at $(0, 0)$



5. $f(x) = \sqrt{x+2}$ at $(-1,\ 1)$



6. $f(x) = -0.5(x-2)^2 + 3$ at $(4, 1)$



Problems 7-10, find the equation of the tangent line to the graph of f at the given point. Use the slopes found for each problem from questions #3-6. Leave your answer in point-slope form.

7. $f(x) = x^2 - 6$ at $(1, -5)$	**8.** $f(x) = x^3 - 3x$ at $(0, 0)$
9. $f(x) = \sqrt{x+2}$ at $(-1, 1)$	**10.** $f(x) = -0.5(x-2)^2 + 3$ at $(4, 1)$

Problems 11-12, use the difference quotient to find the slope of the curve at the given point.

11. $f(x) = x^3 - x$ at $x = 1$

12. $f(x) = \sqrt{4-x}$ at $x = 0$

Problems 13-14, Find the equation of the tangent line and normal line at the given *x*-value.

13. $f(x) = 2x - 3$ at $x = -1$

14. $f(x) = \sqrt{x}$ at $x = 9$

15. Use the limit definition of slope of a curve at a point to find the point on $f(x) = x^2 - 6x + 2$ where the tangent line is horizontal.

Problems 16-20, Given $f(x) = \frac{x}{x-3}$

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16. Use the limit definition to compute the slope of the tangent line at $x = a$.

---

17. Find the slope of the curve at $x = 1$

---

18. Write the equation of the tangent line to the curve at $x = 1$.

---

19. Write the equation of the normal line to the curve at $x = 1$.

---

20. Use the equation from question 18 to estimate the value of $x = 1.1$

2.2 Tangent Lines and the Derivative Homework

Problems 1– 6, Find $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ for each of the functions given below. Write the equation of the tangent line and the normal line to the graph of $f(x)$ at the given value of x.

1. $f(x) = 7 - 3x - x^2$

2. $g(x) = \sqrt{2-x}$

3. Find the equation of the tangent line to the graph of $f(x) = 7 - 3x - x^2$ at $x = 1$

4. Find the equation of the tangent line to the graph of $g(x) = \sqrt{2-x}$ at $x = -7$

5. Find the equation of the normal line to the graph of $f(x) = 7 - 3x - x^2$ at $x = 1$

6. Find the equation of the normal line to the graph of $g(x) = \sqrt{2-x}$ at $x = -7$

Problems 7 – 12, given the function $g(x) = \sqrt{x-2}$

7. Find $g'(x)$ by using the limit process.

8. Find the slope of the tangent line drawn to the graph of $g(x)$ at $x = 4$

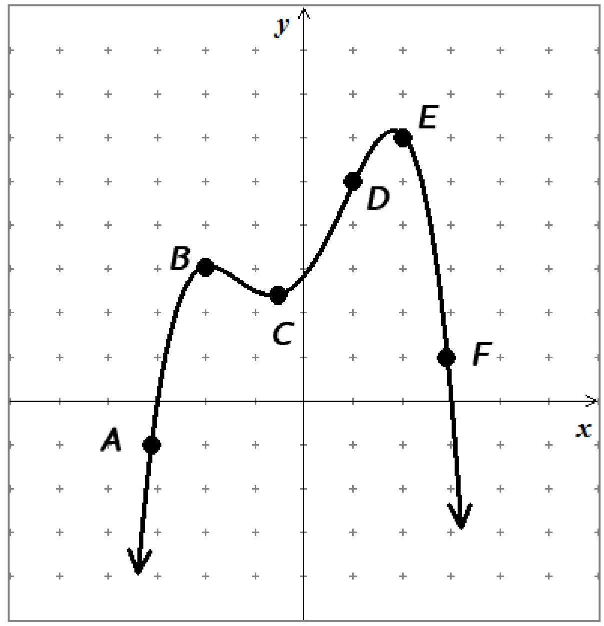
9. Find the slope of the tangent line drawn to the graph of $g(x)$ at $x = 6$

10. Write the equation of the tangent line to the graph of $g(x)$ at $x = 6$.

11. Write the equation of the normal line to the graph of $g(x)$ at $x = 6$.

12. Use the alternate form of the derivative $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$, where $a = 4$.

13. Complete the table below. Estimate the value of $f'(x)$ at the indicated x-values by drawing a tangent line and estimating its slope. Describe the behavior at the given point. Write the equation of the tangent line.



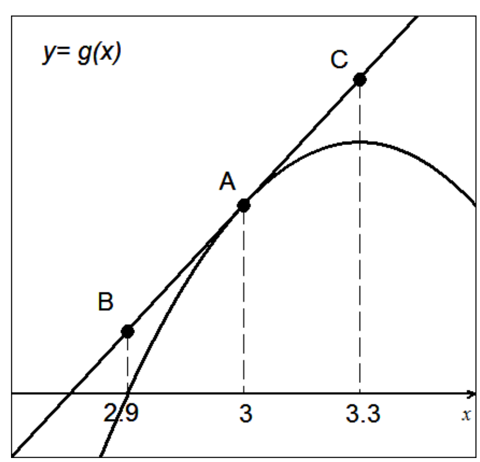
Point of tangency	Behavior at this point? increasing, decreasing, relative maximum or relative minimum	Estimated Slope of Tangent Line	Equation of the tangent line at given point:
A			
B			
C			
D			
E			
F			

Problems 14 – 18, finding a numeric derivative from a table of values at a point can be estimated by finding the slope of the secant line through two points on the graph at either side of the point.

x	-3	-2	-1	0	1	2	4	5	6
$g(x)$	4	3.6	2	-3	-1	0	1	2	3

x-value	Estimate of the derivative	Is the function increasing or decreasing here?	Equation of the tangent line at the value of x
14. $x = -2$			
15. $x = 0$			
16. $x = 1$			
17. $x = 4$			
18. $x = 5$			

19. Given $y = g(x)$ at right, where $g(3) = 18$ and $g'(3) = 2.5$. Find the coordinates of *A, B,* and *C.*



Problems 20 – 23, Each of the following expressions represents the derivative of some function $y = f(x)$ for some value, $x = c$. Identify the function $y = f(x)$ and the value for $x = c$.

20. $\lim_{h \to 0} \frac{7 - 2(2 + h) - 3}{h}$

21. $\lim_{x \to 9} \frac{81 - x^2}{x - 9}$

22. $\lim_{h \to 0} \frac{(-1 + h)^3 + 1}{h}$

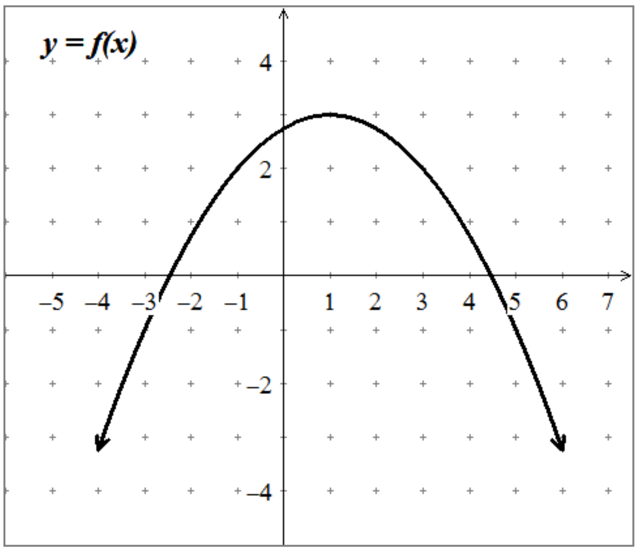
23. $\lim_{x \to -7} \frac{\sqrt{2 - x} + 3}{x + 7}$

24. If $f'(5) = -4$ and $f(5) = 8$, write the equation of the normal line to $y = f(x)$ at $x = 5$.

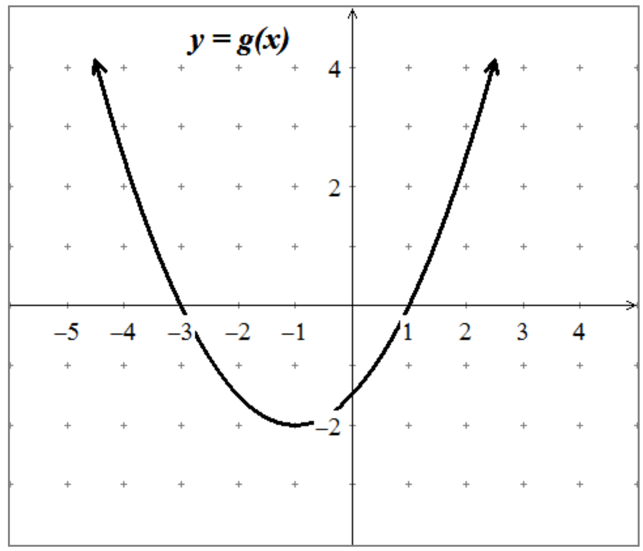
25. Use the alternate form of the derivative to find the equation of the tangent line to the curve at $x = -1$ for the function $f(x) = 2x^2 - 7x + 3$.

Problems 1–2, estimate the slope of the curve at the indicated *x*-coordinate.

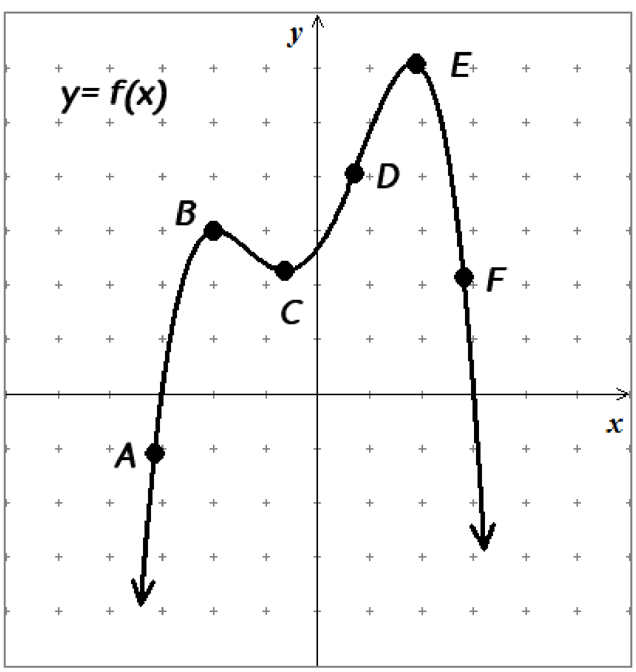
1.
A.) $x = -1$ **B.)** $x = 1$ **C.)** $x = 4$



2.
A.) $x = 0$ **B.)** $x = -2$ **C.)** $x = 1$



Problems 3–7, use the graph of *f* shown below.



3. At which labeled point(s) is the slope of the curve

A. positive

B. negative

C. zero

4. Where is f increasing?

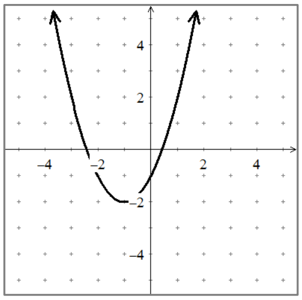
5. Where is f decreasing?

6. Which labeled point has the greatest slope?

7. Which labeled point has the least slope?

Problems 8–15, match each $y = f(x)$ graph with its $f'(x)$ graph A–H.

8.



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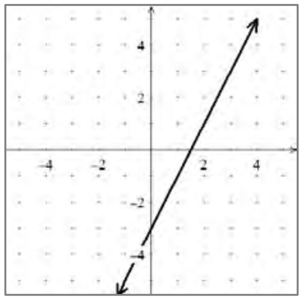

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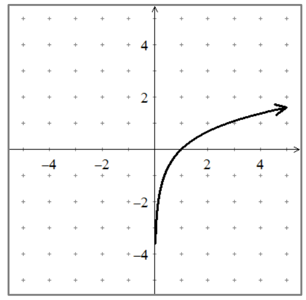

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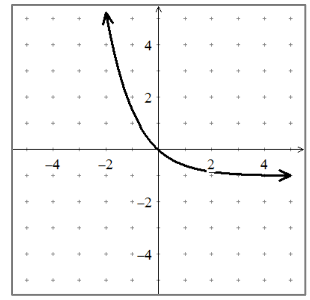

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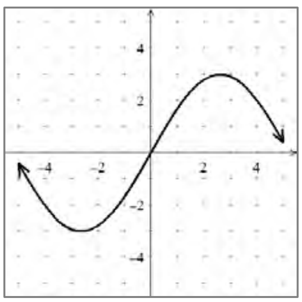

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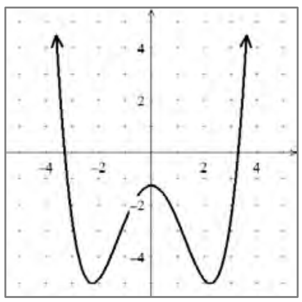

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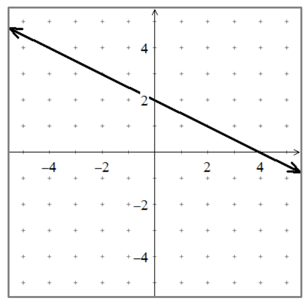

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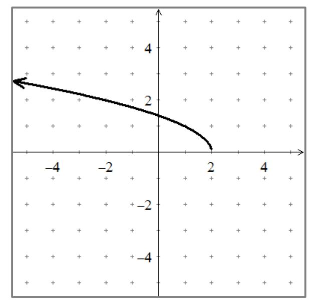
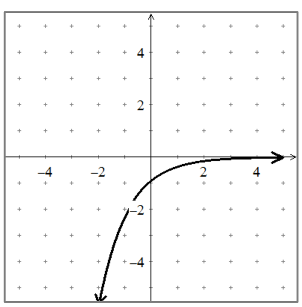


A.



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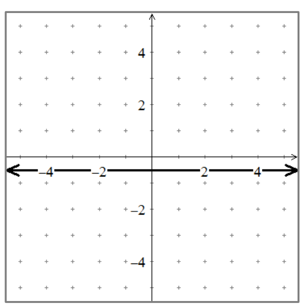
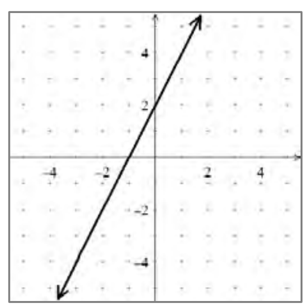


C.



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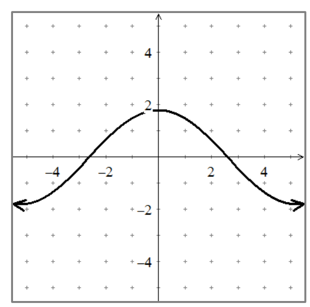

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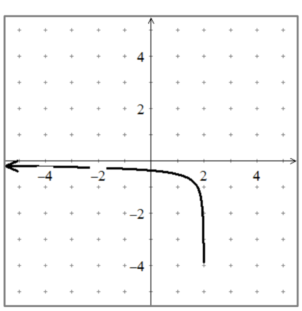

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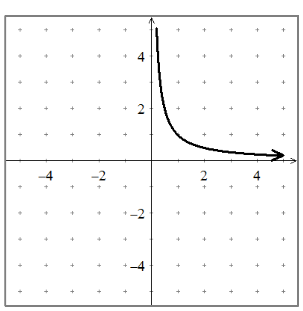

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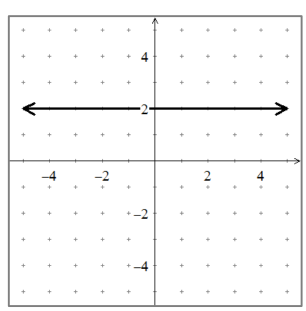

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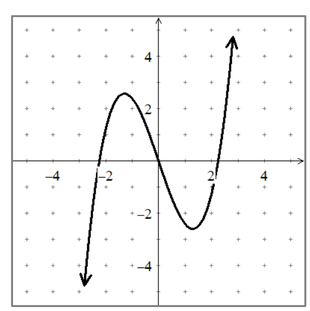
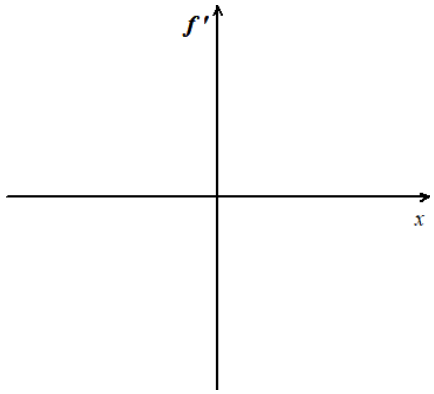
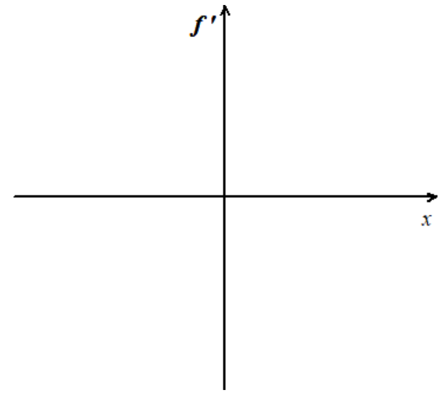


Problems 16–19, sketch functions with the following characteristics.

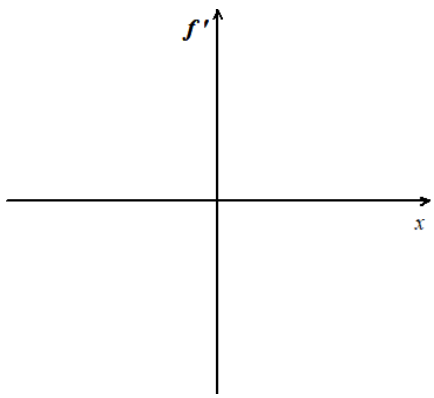
16. The slope is everywhere positive and the function is increasing rapidly.



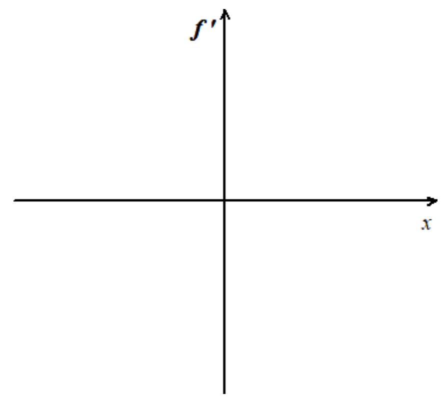
17. The slope is everywhere positive and the function is increasing slowly.



18. The slope is everywhere negative and the function is decreasing rapidly.



19. The slope is everywhere negative and the function is decreasing slowly.



20. Use the table of values below, for a continuous function, $f(x)$, to answer the following questions.

x	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	18	13	10	7	6	6	8	14	17	22	27

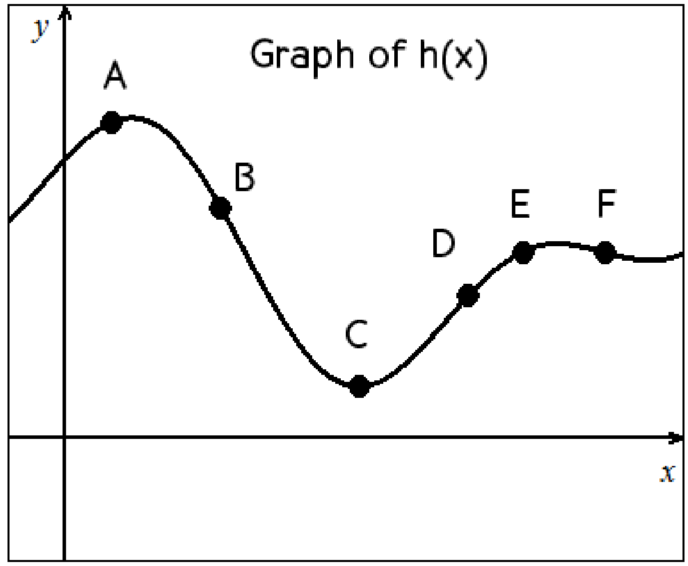
A. Where is the rate of change of $f(x)$ positive?

B. Where is the rate of change of $f(x)$ negative?

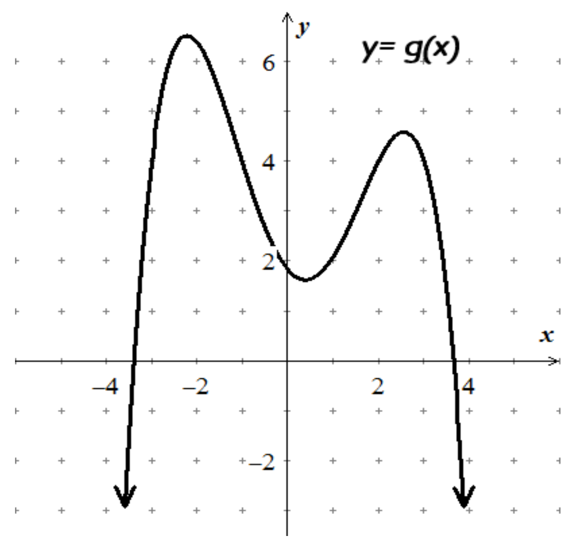
C. Where does the rate of change of $f(x)$ appear to be the greatest?

21. Match the points on the graph of $h(x)$ with the value of $h'(x)$ in the table.

Value of $h'(x)$	**Point on** $h(x)$
-3	
$-\frac{1}{3}$	
0	
$\frac{1}{2}$	
1	
2	



Problems 22–26, use the graph of $g(x)$ below to complete each prompt. Justify your choice.



22. Label a point A on the graph of $y = g(x)$ where the derivative would be negative.

23. Label a point B on the graph of $y = g(x)$ where the value of the function would be negative.

24. Label a point C on the graph of $y = g(x)$ where the derivative is greatest in value.

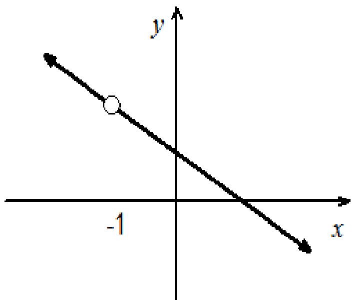
25. Label a point D on the graph of $y = g(x)$ where the derivative is zero.

26. Label two different points F and G, on the graph of $y = g(x)$ where the values of the derivative are opposites.

2.4 Differentiability and Continuity Homework

Problems 1-6, determine if the function $f(x)$ is continuous at the value *c*. If not continuous, state which rule of continuity the function fails.

1.



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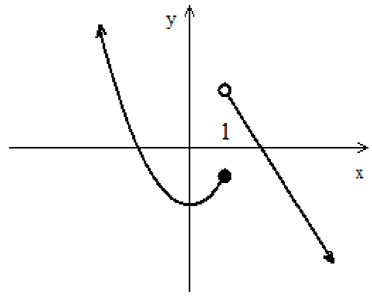

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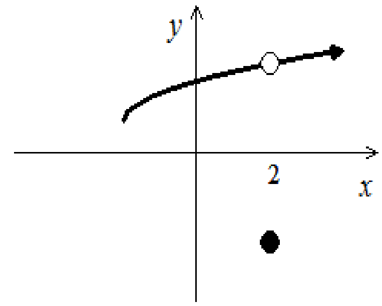

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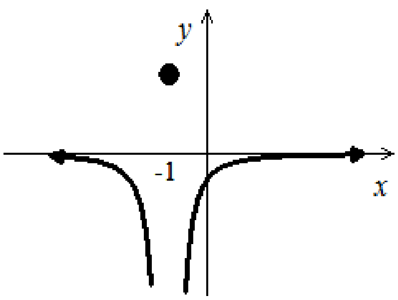

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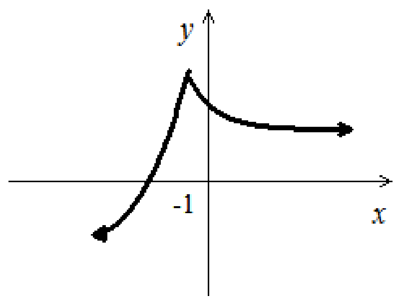

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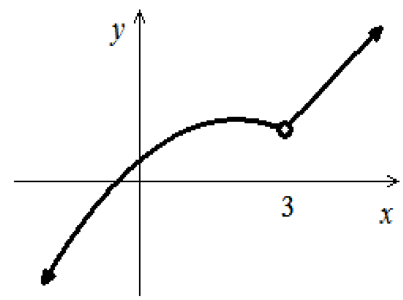


Problems 7-10, state the value(s) of *x* where the function is discontinuous. Justify your reasoning.

7. $f(x) = \dfrac{x^2 + 5x - 24}{x^2 - 64}$

8. $g(x) = \tan x$

9. $h(x) = 2^x + x^2$

10. $f(x) = \dfrac{x^3 - 27}{x - 3}$

Problems 11-12, determine whether the function is continuous at the "transition value," where the function rules change, by using the definition of continuity.

11. $f(x) = \begin{cases} x^2 - 3, & x < 1 \\ 2 - x, & x \geq 1 \end{cases}$

12. $g(x) = \begin{cases} 2(x+2), & x \leq -1 \\ \sqrt{x+1} + 2, & x > -1 \end{cases}$

Problems 13-16, find the value of the constant *k* that makes the function continuous. Show your set up and work.

13. $f(x) = \begin{cases} x^2 + 1, & x < 2 \\ kx - 3, & x \geq 2 \end{cases}$

14. $f(x) = \begin{cases} \sqrt{2x}, & x > 2 \\ kx + 1, & x \leq 2 \end{cases}$

15. $f(x) = \begin{cases} kx + 5, & x < -1 \\ kx^2, & x \geq -1 \end{cases}$

16. $f(x) = \begin{cases} x^2 + 3, & x \geq -2 \\ kx^2 - 2x, & x < -2 \end{cases}$

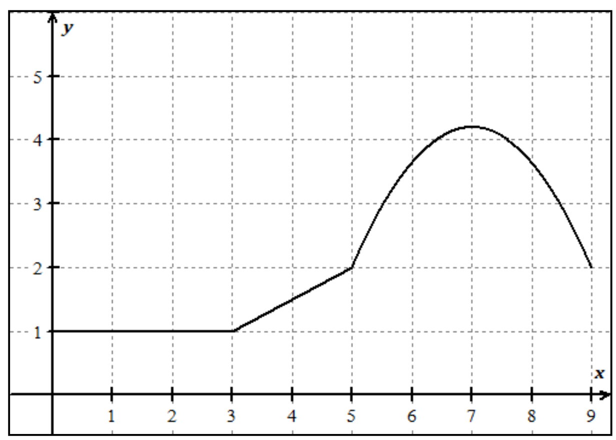
Problems 17 – 18, Use the alternate form of the derivative to determine whether each function is differentiable at the given *x*-value. Then, verify that the function is continuous at $x = c$.

17. $f(x) = \frac{1}{x}$ at $x = 2$

18. $f(x) = x^3 + 3x^2 + 4$ at $x = -1$

Problems 19 – 23: Use the graph of the function *f(x)* shown in the figure at right.

19. $f'(4) =$ ____________________



20. For which values of x is $f'(x) < 0$?

21. Show that $f'(3)$ does not exist.

22. Is f continuous at $x = 5$? Explain why, or why not.

23. Which is larger $f'(5.5)$ or $f'(6.5)$?

Multiple Choice.

24. If $\lim_{x \to 3} f(x) = 9$, which of the following must be true?

I. f is continuous at $x = 3$
II. f is differentiable at $x = 3$
III. $f(3) = 9$

A. None	**B.** II Only	**C.** I and III Only	**D.** I, II, and III

25. At $x = 2$, the function given by $g(x) = \begin{cases} 2x, & x < 2 \\ x^2, & x \geq 2 \end{cases}$ is

A. Differentiable but not continuous
B. Continuous but not differentiable
C. Both continuous and differentiable
D. Neither continuous nor differentiable

Problems 26 – 28: Evaluate each of the following on your calculator.
Explain any incorrect answers.

26. Find $\left.\frac{dy}{dx}\right|_{x=2}$ given $y = \ln x$

27. $f'(-3)$ if $f(x) = \frac{2x^2+1}{4x+5}$

28. $y'(1)$ if $y = \sqrt[3]{x-1}$

2.5 Basic Differentiation Rules Homework

Problems 1- 14, use the rules of differentiation to find the derivative of the function.

1. $y = 14$	**2.** $y = x^9 + 3\tan x$
3. $y = \frac{1}{x^4} - 4\cos x$	**4.** $y = \sqrt[3]{x} + 7e^x$
5. $f(x) = 2x - 7\,Ln\,x$	**6.** $f(x) = 3x^2 + 4x - \,Ln\,x^5$
7. $y = -5t^2 + 4t - 3$	**8.** $g(x) = 9\sin x - 2x^3$
9. $y = 4e^x - \sqrt{x}$	**10.** $s(t) = t^3 + 6t^2 - 2\cos t + 9\sin t$
11. $h(x) = x(x^3 - 1)$	**12.** $f(x) = \cos x - \frac{3}{x^3}$
13. $y = (2x - 3)^2$	**14.** $g(x) = 3e^x + 5\ln x^3 - 2x$

Problems 15-22, Use algebraic techniques to rewrite each function. Then, find the derivative. Answers should have single term denominators when possible.

15. $f(x) = 2\sin x + \frac{1}{x^2}$

16. $y = \frac{5x^3+7x^2}{x}$

17. $y = \frac{3e^x + 2e^{2x}}{e^x}$

18. $g(x) = \frac{x^3+5x^2+2}{x^2}$

19. $f(x) = \sqrt{x} + 9\sqrt[3]{x}$

20. $g(x) = x^{2/3} + x^{1/3} + 2\sin x$

21. $h(x) = \frac{x - 6\sqrt{x}}{3x^2}$

22. $y = \frac{x^3 - 8}{x-2}$

23. Find k so that the function $f(x) = x^2 + kx$ will be tangent to the line $y = 2x - 9$

Problems 24-29, Use trigonometric, exponential, and logarithmic derivative rules to find the derivative. Answers should have no negative exponents.

24. $f(x) = 5\sin x - \ln x^2$	**25.** $h(x) = ln(2x^3) - ln\left(\sqrt{5x^3}\right)$
26. $y = \ln(\pi x^3) - \frac{2}{\sqrt{x^3}}$	**27.** $g(x) = \ln\left(\frac{\sqrt{x}}{x^\pi}\right)$
28. $f(x) = 4\,tan\,\pi + 2\cos x - \ln x^2 - 3e^x$	**29.** $y = 5e^2 - 2e^x + \sin x - \cos\pi$

Problems 30 – 31, find the equation of the tangent line to each graph at the point $(1, 0)$.

30. $y = \ln x^2$	**31.** $y = \ln\sqrt{x^3}$

Problems 32 – 33, Find the points on the graph of $f(x)$ where the tangent line is horizontal.

32. $f(x) = 12x - x^3$

33. $g(x) = 4x^{-2} + 8x + 1$

Problems 34 – 35, Find an equation of the tangent line to the graph at $x = a$.

34. $f(x) = 6x - 16\sqrt{x}$ **;** $x = 4$

35. $g(x) = \sin x + \cos x$ **;** $x = \frac{3\pi}{4}$

Problems 36 – 37, Solve.

36. Find the values of a and b for $h(x) = x^2 + ax + b$ so $h(1) = 0$ and $h'(1) = 4$.

37. Find an equation of the line perpendicular to the tangent to the curve $y = x^3 - 3x + 4$ at $x = 2$.

2.6 Product and Quotient Rules Homework

Problems 1 – 8, Use the Product or Quotient Rule to differentiate the function.

1. $y = (x^2 - 3x)(x^2 + 1)$

2. $s(t) = (\sqrt{t} + 3)\,(4 - t^3)$

3. $f(x) = x^4 \sin x$

4. $s(\theta) = \frac{\cot\theta}{\sqrt{\theta}}$

5. $h(x) = \frac{3x^2}{x^3 - 1}$

6. $g(x) = \frac{2x - 3}{5x + 2}$

7. $y = \frac{x^2}{\cos x}$

8. $g(x) = \frac{x \sin x}{x + 2}$

Problems 9 – 10, Find the derivative by rewriting each function. Do not use the Quotient Rule.

9. $f(x) = \frac{2x^2 - 3x + 5}{\sqrt{x}}$

10. $g(x) = \frac{\sin x - x^3}{x^2}$

Problems 11-14, Find the derivative of the function, $f'(x)$ at the specified point $x = c$. Then, write the equation of the tangent line at the point.

11. $f(x) = (x^3 + 1)(x^2 - 2x + 1)$ **;** $c = -1$

12. $g(x) = \frac{x^2 - 9}{x - 2}$ **;** $c = 1$

13. $f(\theta) = \theta \cos\theta \;;\; c = \frac{\pi}{2}$

14. $h(x) = (3e^x - x^2)(x^3 + 1); c = 0$

Problems 15 – 18, Find the derivative.

15. $g(x) = \frac{\pi}{1+e^x}$

16. $y = \frac{e^x}{x^2 + 1}$

17. $f(x) = \sqrt{x}\left(2x^4 - 5x + x^{-\frac{1}{2}}\right)$

18. $y = \frac{1}{2x^3 - x}$

Problems 19 – 22, Suppose *u* and *v* are functions of *x* that are differentiable at $x = 1$, and that $u(1) = 4$, $u'(1) = -3$, $v(1) = -2$, and $v(1) = 6$. Find the values of the derivatives at $x = 1$.

19. $\frac{d}{dx}[uv]$

20. $\frac{d}{dx}\left[\frac{u}{v}\right]$

21. $\frac{d}{dx}\left[\frac{v}{u}\right]$

22. $\frac{d}{dx}[3uv - 2v]$

Problems 23 – 26, Let $f(x)$ and $g(x)$ be differentiable functions with the following values:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-4	1	2	-1
0	1	-2	1	3
2	-3	-1	-1	1

23. If $h(x) = 3\,f(x)\,g(x)$, find $h'(-1)$.

24. Find the slope of the tangent line to $k(x) = g(x) \cdot \cos x$ when $x = 0$.

25. If $s(x) = \frac{-2 \cdot g(x)}{f(x)}$, find $s'(-1)$. Write the equation of the tangent line at $x = -1$.

26. If $k(x) = [g(x) - 1][2f(x) + 3]$, find $k'(2)$. Write the equation of the tangent line at $x = 2$.

2.7 Velocity and Other Rates Of Change Homework

Problems 1 – 2, Interpret each scenario.

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1. The temperature T, in degrees Fahrenheit, of your holiday turkey placed into a hot oven is modeled by $T = f(t)$, where t is the time in minutes since the turkey was put in the oven.

A. What is the sign of $f'(t)$? Explain how you know.

B. What are the units of $f'(35)$?

C. Explain the meaning of the statement $f(35) = 106$ in the context of the problem.

D. What is the meaning of the statement $f'(35) = 3$?

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2. A skateboard manufacturer wants to understand how the price of their longboard affects the overall sales. Suppose that at a price of p dollars, the quantity, Q, of longboards sold is a function of the longboards price, that is $Q = f(p)$**.**

A. Explain the meaning of $f(175) = 80{,}000$ in a complete sentence.

B. Using the correct units, explain the meaning of $f'(175) = -45$

C. If the rate from part (B) holds true for $150 \leq p \leq 190$, how many longboard are predicted to sell when the price of a longboard is $190 ? Show the work leading to your answer.

Problems 3 – 6, Solve.

3. A child is tossing a small ball into the air. The position of the ball at a time t, seconds after it is thrown into the air is given by $s(t) = -5.2t^2 + 25t + 4$ feet. Find $s'(4)$, explain the meaning of the value you found. Is the ball speeding up or slowing down? How do you know?

4. To estimate the height of a building, a stone is dropped into a fountain at ground level from the top of the building. How high is the building, in feet, if the splash is seen 5.6 seconds after the stone is dropped?

5. Derivative Airlines can calculate the total dollar cost of a flight with x passengers using the model $C(x) = 0.0005x^3 - 0.4x^2 + 125x$.

A. Estimate the marginal cost (that is, the cost of producing one additional unit) of an additional passenger if the flight already has 160 passengers.

B. Compare your estimate to the actual cost of an additional passenger.

C. Is it more expensive to add a passenger when $x = 160$ or when $x = 180$?

7. Randy the Roach is moving in a smooth and continuous manner along a line. His location has been tracked and his position at any given time, $0 \leq t \leq 5$, is given in the table below. [*Note, this prompt has no units.*]

t	0	0.5	1	1.8	2	2.4	3	3.5	4	5
$s(t)$	25	28.2	31	37	43	39	34	21	17	2

A. Find the displacement of Randy Roach during the given interval $[0, 5]$. Show work that leads to your answer.

B. What is the minimum number of times Randy changes direction? Explain your answer.

C. What is Randy's average velocity for $[0, 2]$? Show work that leads to your answer.

D. Estimate Randy's velocity at $t = 2.4$. Use proper notation and show work.

E. Is it possible to determine the time and position when Randy is farthest away from the origin? Why or why not?

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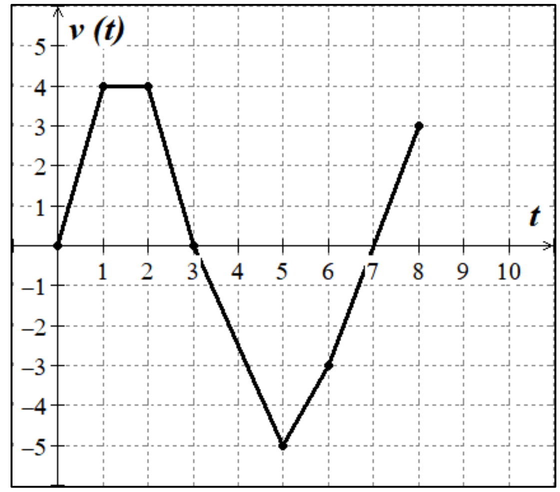
8. An object moves along a horizontal line according to $s(t) = t^3 - 9t^2 + 24t$.

A. When is s increasing, and when is it decreasing? Show work that supports your answer.

B. Find the total distance traveled in the first 5 seconds of motion. Show work.

9. The graph at right shows the velocity $v(t)$ of a particle, in ft/sec, moving along a horizontal line for $0 \le t \le 8$.



A. On what open intervals or at what time $0 < t < 8$ is the particle at rest? Justify.

B. On what open intervals $0 < t < 8$ is the particle moving to the right? Justify.

C. On what open intervals or at what time(s) $0 < t < 8$ is the particle moving at its greatest speed? Greatest velocity?

D. On what open intervals or at what time(s) $0 < t < 8$ is the particle's speed increasing? Decreasing? Justify.

E. What is the particle's acceleration at $t = 4.5$ seconds? Explain what this number means in terms of the particle's velocity?

F. On what open intervals or at what time(s) $0 < t < 8$ is acceleration of the particle the greatest?

G. Find the displacement of the particle during the time interval $[0, 3]$. Justify.