

Unit 2: Differentiation – Definition and Fundamental Properties

| DAY | LESSON & OBJECTIVES | ASSIGNMENTS |
|-----|---|-------------|
| 1 | 2.1 Rates of Change and the Tangent Line Problem <ul style="list-style-type: none"> • Interpret rates of change over an interval or at a single point by graphical and numerical methods • Write equations of tangent lines and normal lines | |
| 2 | 2.2 Tangent Lines and the Derivative <ul style="list-style-type: none"> • Use limit definition of the derivative, proper notation to find the general form of a derivative • Analyze function behavior using the derivative | |
| 3 | 2.3 Understanding the Derivative and Linear Approximations <ul style="list-style-type: none"> • Explore the concept of local linearity • Use the tangent line to investigate function behavior • Estimate derivatives graphically and from tables | |
| 4 | | |
| 5 | 2.4 Differentiability and Continuity <ul style="list-style-type: none"> • Understand the relationship between differentiability and continuity | |
| 6 | | |
| 7 | AP STYLE QUIZ | |
| 8 | 2.5 Basic Differentiation Rules <ul style="list-style-type: none"> • Find derivatives using basic rules of differentiation for polynomial, power, sine, cosine, tangent, exponential and logarithmic functions | |
| 9 | | |
| 10 | 2.6 Product and Quotient Rules <ul style="list-style-type: none"> • Apply the product and quotient rules algebraically for polynomial, power, sine, cosine, tangent, exponential and logarithmic functions | |
| 11 | | |
| 12 | 2.7 Velocity and Other Rates of Change <ul style="list-style-type: none"> • Solve basic rectilinear motion problems involving position, speed, velocity and acceleration • Interpret the effect of a one-unit change | |
| 13 | | |
| 14 | AP STYLE TEST #2 | |
| 15 | | |

Unit 2: Differentiation: Definition and Basic Derivative Rules

| MATHEMATICAL PRACTICES | | |
|---------------------------------|-------------------------------------|---|
| TOPICS | PRACTICE | SKILL |
| 2.1 | Connecting Representations | 2.B: Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations. |
| 2.3; 2.5; 2.6; 2.7; 2.8; 2.9 | Implementing Mathematical Processes | 1.E: Apply appropriate mathematical rules or procedures, with and without technology. |
| 2.2; 2.10 | Connecting Representations | 1.D: Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., <i>rate of change and accumulation</i>) or processes (e.g., <i>differentiation and its inverse process, anti-differentiation</i>) to solve problems. |
| 2.2 | Communication and Notation | 4.C: Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y' , and $\frac{dy}{dx}$) |
| 2.4 | Justification | 3.E: Provide reasons or rationales for solutions or conclusions. |

Unit 2: Differentiation: Definition and Basic Derivative Rules

| TOPIC | Lesson Topic | Learning Objectives | Essential Knowledge | Mathematical Practices |
|-------|---|---------------------|--|------------------------|
| 2.1 | Defining Average and Instantaneous Rates of Change at a Point | CHA-2.A CHA-2.B | CHA-2.A.1 CHA-2.B.1 | 2.B |
| 2.2 | Defining the Derivative of a Function and Using Derivative Notation | CHA-2.B CHA-2.C | CHA-2.B.2 CHA-2.B.3 CHA-2.B.4 CHA-2.C.1 | 1.D 4.C |
| 2.3 | Estimating Derivatives of a Function at a Point | CHA-2.D | CHA-2.D.1 CHA-2.D.2 | 1.E |
| 2.4 | Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist | FUN-2.A | FUN-2.A.1 FUN-2.A.2 | 3.E |
| 2.5 | Applying the Power Rule | FUN-3.A | FUN-3.A.1 | 1.E |
| 2.6 | Derivative Rules: Constant, Sum, Difference, and Constant Multiple | FUN-3.A | FUN-3.A.2 FUN-3.A.3 | 1.E |
| 2.7 | Derivatives of $\cos x$, $\sin x$, e^x , and $\ln x$ | FUN-3.A LIM-3.A | FUN-3.A.4 LIM-3.A.1 | 1.E |
| 2.8 | The Product Rule | FUN-3.B | FUN-3.B.1 | 1.E |
| 2.9 | The Quotient Rule | FUN-3.B | FUN-3.B.2 | 1.E |
| 2.10 | Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions | FUN-3.B | FUN-3.B.3 | 1.D |

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| | AB | BC |
|---|---|-------------|
| AP EXAM WEIGHTING | 10% - 12% | 4% - 7% |
| PACING GUIDE | 13 - 14 days | 9 - 10 days |
| Topic 2.1 | Defining Average and Instantaneous Rates of Change at a Point | |
| Enduring Understanding | CHA-2: Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. | |
| Learning Objective | Essential Knowledge | |
| <p>CHA-2.A: Determine average rates of change using difference quotients.</p> <p>CHA-2.B: Represent the derivative of a function as the limit of a difference quotient.</p> | <p>CHA-2.A.1: The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.</p> <p>CHA-2.B.1: The instantaneous rate of change of a function at $x = a$ can be expressed by $\lim_{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$ or $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{x-a}$ provided the limit exists. These are equivalent forms of the definition of the derivative and are denoted $f'(a)$.</p> | |
| Topic 2.2 | Defining the Derivative of a Function and Using Derivative Notation | |
| Enduring Understanding | CHA-2: Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. | |
| Learning Objective | Essential Knowledge | |
| <p>CHA-2.B: Represent the derivative of a function as the limit of a difference quotient.</p> <p>CHA-2.C: Determine the equation of a line tangent to a curve at a given point.</p> | <p>CHA-2.B.2: The derivative of f is the function whose value at x is $\lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$, provided this limit exists.</p> <p>CHA-2.B.3: For $y = f(x)$, notations for the derivative include $\frac{dy}{dx}$, $f'(x)$, and y'.</p> <p>CHA-2.B.4: The derivative can be represented graphically, numerically, analytically, and verbally.</p> | |

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| Topic 2.3 | Estimating Derivatives of a Function at a Point |
| Enduring Understanding | CHA-2: Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals. |
| Learning Objective | Essential Knowledge |
| CHA-2.D: Estimate derivatives. | CHA-2.D.1: The derivative at a point can be estimated from information given in tables or graphs. CHA-2.D.2: Technology can be used to calculate or estimate the value of a derivative of a function at a point. |
| Topic 2.4 | Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist |
| Enduring Understanding | FUN-2: Recognizing that a function's derivative may also be a function allows us to develop knowledge about the related behaviors of both. |
| Learning Objective | Essential Knowledge |
| FUN-2.A: Explain the relationship between differentiability and continuity. | LIM-2.A.1: If a function is differentiable at a point, then it is continuous at that point. In particular, if a point is not in the domain of f , then it is not in the domain of f' . LIM-2.A.2: A continuous function may fail to be differentiable at a point in its domain. |
| Topic 2.5 | Applying the Power Rule |
| Enduring Understanding | FUN-3: Recognizing opportunities to apply derivative rules can simplify differentiation. |
| Learning Objective | Essential Knowledge |
| FUN-3.A: Calculate derivatives of familiar functions. | FUN-3.A.1: Direct application of the definition of the derivative and specific rules can be used to calculate the derivative for functions of the form $f(x) = x^r$. |

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| Topic 2.6 | Derivative Rules: Constant, Sum, Difference, and Constant Multiple |
| Enduring Understanding | FUN-3: Recognizing opportunities to apply derivative rules can simplify differentiation. |
| Learning Objective | Essential Knowledge |
| FUN-3.A: Calculate derivatives of familiar functions. | <p>FUN-3.A.2: Sums, differences, and constant multiples of functions can be differentiated using derivative rules.</p> <p>FUN-3.A.3: The power rule combined with sum, difference, and constant multiple properties can be used to find the derivative for polynomial functions.</p> |
| Topic 2.7 | Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$ |
| Enduring Understanding | FUN-3: Recognizing opportunities to apply derivative rules can simplify differentiation. |
| Learning Objective | Essential Knowledge |
| FUN-3.A: Calculate derivatives of familiar functions. | FUN-3.A.4: Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions. |
| Enduring Understanding | LIM-3: Reasoning with definitions, theorems, and properties can be used to determine a limit. |
| LIM-3.A: Interpret a limit as a definition of a derivative. | LIM-3.A.1: In some cases, recognizing an expression for the definition of the derivative of a function whose derivative is known offers a strategy for determining a limit. |

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| Topic 2.8 | The Product Rule |
| Enduring Understanding | FUN-3: Recognizing opportunities to apply derivative rules can simplify differentiation. |
| Learning Objective | Essential Knowledge |
| FUN-3.B: Calculate derivatives of products and quotients of differentiable functions. | FUN-3.B.1: Derivatives of products of differentiable functions can be found using the product rule. |
| Topic 2.9 | The Quotient Rule |
| Enduring Understanding | FUN-3: Recognizing opportunities to apply derivative rules can simplify differentiation. |
| Learning Objective | Essential Knowledge |
| FUN-3.B: Calculate derivatives of products and quotients of differentiable functions. | FUN-3.B.2: Derivatives of quotients of differentiable functions can be found using the quotient rule. |
| Topic 2.10 | Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions |
| Enduring Understanding | FUN-3: Recognizing opportunities to apply derivative rules can simplify differentiation. |
| Learning Objective | Essential Knowledge |
| FUN-3.B: Calculate derivatives of products and quotients of differentiable functions. | FUN-3.B.3: Rearranging tangent, cotangent, secant, and cosecant functions using identities allows differentiation using derivative rules. |