CALCULUS BC
WORKSHEET 1 ON DIFFERENTIAL EQUATIONS

Work the following on **notebook paper**. Do not use your calculator.

Solve for y as a function of x.

1. $\frac{dy}{dx} = \frac{x-3}{y}$ and $y(2) = -5$

2. $y' = 2x\sqrt{y}$ and $y(2) = 25$

3. $\frac{dy}{dx} = 4y^2 \sec^2(2x)$ and $y\left(\frac{\pi}{8}\right) = 1$

4. $xy\frac{dy}{dx} = \ln x$ and $y(1) = 2$

5. $y' = 2x \sec y$ and $y(2) = -\frac{\pi}{2}$

6. $y' - xe^y = 2e^y$ and $y(0) = 0$

7. $\frac{dy}{dx} = 2xy^3 \sin(x^2)$ and $y(0) = -1$

8. $\frac{dy}{dx} = \frac{1}{y^2}$ and $y(0) = 4$

9. Find a curve in the xy-plane that passes through the point (0, 3) and whose tangent line at a point (x, y) has slope $\frac{2x}{y^2}$.

Write a differential equation to represent the following:

10. The rate of change of a population y, with respect to time t, is proportional to t.

11. The rate of change of a population P, with respect to time t, is proportional to the cube of the population.

12. Let $P(t)$ represent the number of wolves in a population at time t years, where $t \geq 0$. The rate of change of the population $P(t)$, with respect to t, is directly proportional to $500 - P(t)$.

13. Water leaks out of a barrel at a rate proportional to the square root of the depth of the water at that time.

14. Oil leaks out of a tank at a rate inversely proportional to the amount of oil in the tank.

Answers to Worksheet 1

1. $y = -\sqrt{x^2 - 6x + 33}$`
2. $y = \left(\dfrac{x^2}{2} + 3\right)^2$
3. $y = \dfrac{1}{3 - 2\tan(2x)}$
4. $y = \sqrt{(\ln x)^2 + 4}$
5. $y = \arcsin\left(x^2 - 5\right)$
6. $y = -\ln\left|1 - 2x - \dfrac{x^2}{2}\right|$
7. $y = -\sqrt{\dfrac{1}{2\cos\left(x^2\right) - 1}}$
8. $y = \sqrt[3]{3x + 64}$
9. $y = \sqrt[3]{3x + 27}$
10. $\dfrac{dy}{dt} = kt$
11. $\dfrac{dP}{dt} = kP^3$
12. $\dfrac{dP}{dt} = k(500 - P)$
13. $\dfrac{dy}{dt} = k\sqrt{y}$
14. $\dfrac{dV}{dt} = \dfrac{k}{V}$

CALCULUS BC
WORKSHEET 2 ON DIFFERENTIAL EQUATIONS

Work the following on **notebook paper**. Do not use your calculator.

Solve for y as a function of x.

1. $\frac{dy}{dx} = 6x^2 y$ and $y(0) = 4$

2. $\frac{dy}{dx} = \frac{1+x}{\sqrt{y}}$ and $y(2) = 9$

3. $\frac{dy}{dx} = (y-2)^2 \cos(3x)$ and $y\left(\frac{\pi}{2}\right) = 3$

4. $\frac{dy}{dx} = 3x + xy$ and $y(4) = -2$

5. $\frac{dy}{dx} = \frac{y-3}{x^2}$, $x \neq 0$, and $y(4) = 0$

6. $\frac{dy}{dx} = x^2 y + 2x^2$, $y(-1) = 4$

---

7. If $\frac{dy}{dx} = y \cos x$ and $y = 3$ when $x = 0$, then y = ?

8. Find an equation of the curve that satisfies $\frac{dy}{dx} = 4x^3 y$ and whose y-intercept is 7.

Multiple choice. Solve. All steps must be shown.

9. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

---

10. Consider the differential equation $\frac{dy}{dx} = xy^3$. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = -1$, and state its domain.

11. Consider the differential equation $\frac{dy}{dx} = \frac{y+5}{x}$, $x \neq 0$. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-3) = 1$, and state its domain.

TURN->>>

12. The rate at which a population of bears in a national forest grows is proportional to $600 - B(t)$, where t is the time in years and $B(t)$ is the number of bears. At time $t = 0$, there are 200 bears in the forest. The rate of change of the population of bears is modeled by the differential equation $\frac{dB}{dt} = \frac{1}{2}(600 - B)$, and $y = B(t)$ is the solution to the differential equation with initial condition $B(0) = 200$.

(a) Is the number of bears growing faster when there are 300 bears or when there are 400 bears? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use your answer to determine whether the graph of B is concave up or concave down when there are 450 bears in the forest.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 200$.

Answers to Worksheet 1

1. $y = 4e^{2x^3}$
2. $y = \left(\frac{3}{2}x + \frac{3}{4}x^2 + 21\right)^{2/3}$
3. $y = 2 + \frac{3}{2 - \sin(3x)}$
4. $y = -3 + e^{\frac{x^2}{2} - 8}$
5. $y = 3 - 3e^{\frac{1}{4} - \frac{1}{x}}$
6. $y = -2 + 6e^{\frac{x^3+1}{3}}$
7. $y = 3e^{\sin x}$
8. $y = 7e^{x^4}$
9. $y = \frac{1}{1-2x}$. When $x = 3$, $y = -\frac{1}{3}$. B
10. $y = -\frac{1}{\sqrt{5-x^2}}$, Domain $= \left\{x : -\sqrt{5} < x < \sqrt{5}\right\}$
11. $y = -\frac{1}{\sqrt{5-x^2}}$, Domain $= \left\{x : -\sqrt{5} < x < \sqrt{5}\right\}$

12. (a) The number of bears is growing faster when there are 300 bears since $\frac{dB}{dt}$ is greater when t = 300 bears than when t = 400 bears.

(b) When B = 450 bears. $\frac{d^2B}{dt^2} = -\frac{1}{4}(150) < 0$ so the graph of B is concave down when B = 450 bears.

(c) $B = 600 - 400e^{-\frac{1}{2}t}$

CALCULUS
WORKSHEET ON APPLICATIONS OF DIFFERENTIAL EQUATIONS

Work the following on **notebook paper**. Use your calculator and give decimal answers correct to three decimal places.

1. A pie is removed from an oven at 450° and left to cool at a room temperature of 70°. After 30 minutes, the pie's temperature is 200°. How many minutes after being removed from the oven will the temperature of the pie be 100°?

2. A certain population increases at a rate proportional to the square root of the population. If the population goes from 2500 to 3600 in five years, what is the population at the end of t years?

3. Water leaks out of a barrel at a rate proportional to the square root of the depth of the water at that time. If the water level starts at 36 in. and drops to 35 in. in one hour, how long will it take for all of the water to leak out of the barrel?

4. A student studying a foreign language has 50 verbs to memorize. The rate at which the student can memorize these verbs is proportional to the number of verbs remaining to be memorized, $50 - y$, where the student has memorized y verbs. Assume that initially no verbs have been memorized and suppose that 20 verbs are memorized in the first 30 minutes.
(a) How many verbs will the student memorize in two hours?
(b) After how many hours will the student have only one verb left to memorize?

5. Let $P(t)$ represent the number of wolves in a population at time t years, where $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.
(a) If $P(0) = 500$, find $P(t)$ in terms of t and k.
(b) If $P(2) = 700$, find k.
(c) Find $\lim_{t \to \infty} P(t)$.

Solve. Do not use your calculator.

6. $y' - xy^2 = 0$ and $y(1) = 1$

7. $\dfrac{dy}{dx} - 2xy = 2x$ and $y(0) = 3$

8. $\dfrac{dy}{dx} = \dfrac{3x}{y}$ and $y(1) = -3$

9. $2x\dfrac{dy}{dx} - \ln\left(x^2\right) = 0$ and $y(1) = 3$

Answers to Worksheet on Applications of Differential Equations

1. 71.011 minutes

2. $y=(2t+50)^2$

3. 71.496 hr

4. (a) 43.520 so 44 verbs (b) 3.829 hr (or 229.427 min.)

5. (a) $P=800-\dfrac{300}{e^{kt}}$

   (b) $\dfrac{1}{2}\ln 3$

   (c) 800 wolves

6. $y=\dfrac{2}{3-x^2}$

7. $y=-1+4e^{x^2}$

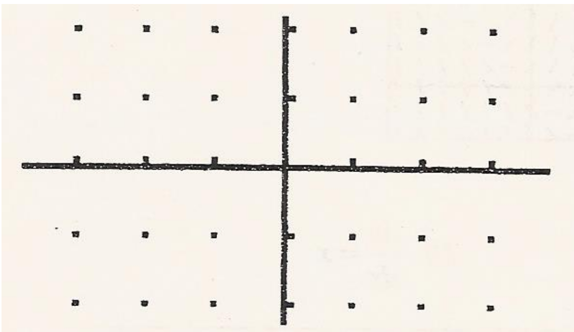
8. $y=-\sqrt{3x^2+6}$

9. $y=\dfrac{(\ln x)^2}{2}+3$

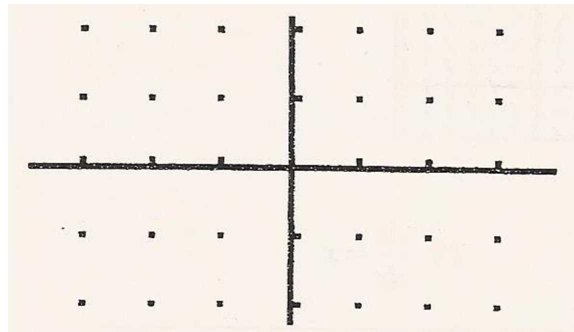
CALCULUS
WORKSHEET ON SLOPE FIELDS

Draw a slope field for each of the following differential equations.

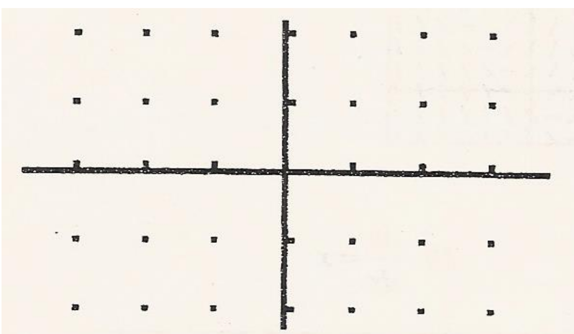
1. $\frac{dy}{dx} = x+1$



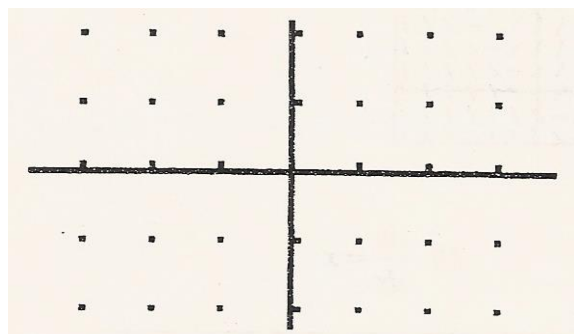
2. $\frac{dy}{dx} = 2y$



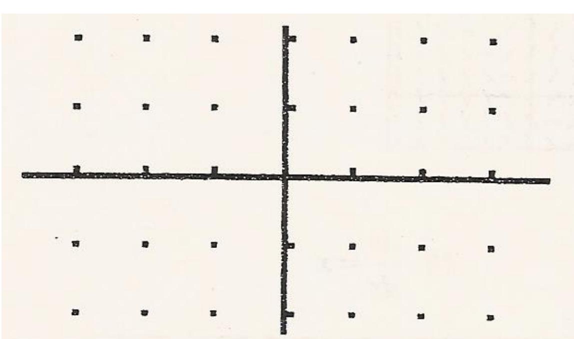
3. $\frac{dy}{dx} = x+y$



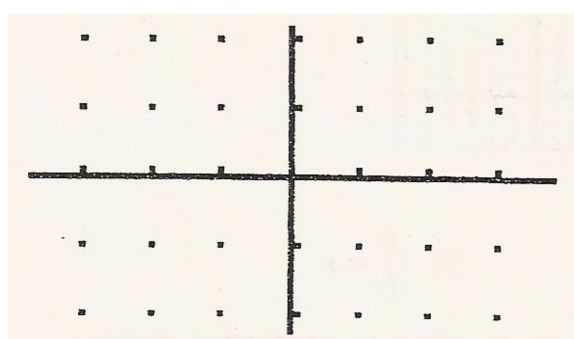
4. $\frac{dy}{dx} = 2x$



5. $\frac{dy}{dx} = y-1$

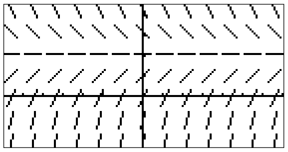


6. $\frac{dy}{dx} = -\frac{y}{x}$

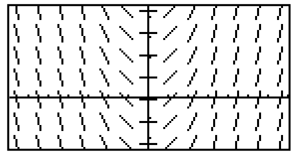


Match the slope fields with their differential equations.

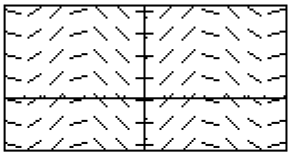
(A)



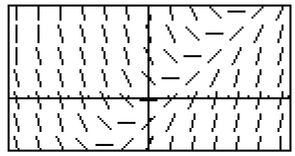
(B)



(C)



(D)



7. $\frac{dy}{dx} = \sin x$

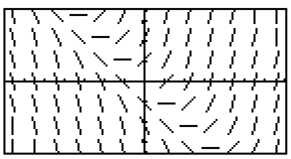
8. $\frac{dy}{dx} = x - y$

9. $\frac{dy}{dx} = 2 - y$

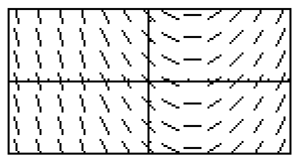
10. $\frac{dy}{dx} = x$

Match the slope fields with their differential equations.

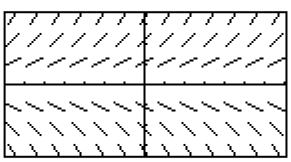
(A)



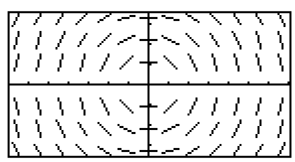
(B)



(C)



(D)



11. $\frac{dy}{dx} = .5x - 1$

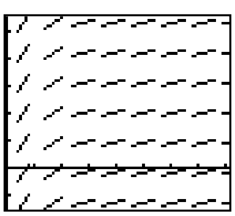
12. $\frac{dy}{dx} = .5y$

13. $\frac{dy}{dx} = -\frac{x}{y}$

14. $\frac{dy}{dx} = x + y$

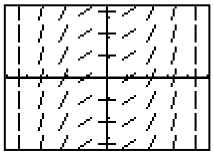
---

15. (From the AP Calculus Course Description)



The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$

16. 

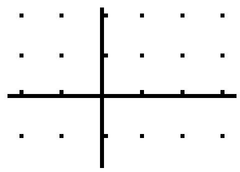
The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

---

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$

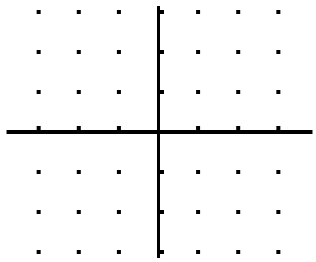
(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(d) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part (c). Was your estimate from part (b) an underestimate or an overestimate? Explain.

TURN->>>

18. Consider the differential equation given by $\frac{dy}{dx} = \frac{x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



(b) Sketch a solution curve that passes through the point (0, 1) on your slope field.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = 1$.

(d) Sketch a solution curve that passes through the point $(0, -1)$ on your slope field.

(e) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(0) = -1$.

Answers to Worksheet on Slope Fields

1. – 6. Graphs

7. C	8. D	9. A	10. B
11. B	12. C	13. D	14. A
15. E	16. D		

17. (a) graph

(b) $y = 1 + \frac{1}{2}(x - 1)$, 1.1

(c) $y = e^{\frac{x^2 - 1}{4}}$, 1.116

(d) underestimate

18. (a) and (b) graphs

(c) $y = \sqrt{x^2 + 1}$

(d) $y = -\sqrt{x^2 + 1}$

CALCULUS BC
WORKSHEET ON EULER'S METHOD

Work the following on notebook paper, showing all steps.

1. (a) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps. Sketch your solution.

   (b) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

   (c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

2. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that $f(2) = 5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5		

3. The curve passing through (2, 0) satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

4. The table gives selected values for the derivative of a function f on the interval $-2 \le x \le 2$. If $f(-2) = 3$ and Euler's method with a step-size of 1.5 is used to approximate $f(1)$, what is the resulting approximation?

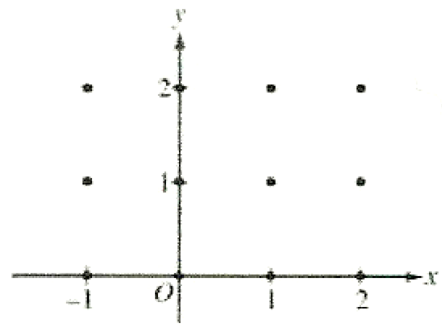
x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

5. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.6)$.

TURN->>>

6. (2005 BC 4)

Consider the differential equation $\frac{dy}{dx} = 2x - y$.



(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point (0, 1).

(b) The solution curve that passes through the point (0, 1) has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

---

7. (Modified version of 2009 BC 4)

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2 y$. Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(-1) = 2.$.

(a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 2$.

Answers to Worksheet on Euler's Method

1. (a) $5\frac{1}{4}$

   (b) $5\frac{1}{2}$

   (c) Error = $\frac{1}{4}$. Use smaller steps.

3. 8.25
4. 2.4
5. 0.25
6. See AP Central.
7. See AP Central (The original part (b) is omitted, and the original part (c) is now part (b)).