CALCULUS AB
WORKSHEET ON SECOND FUNDAMENTAL THEOREM AND REVIEW

Work the following on notebook paper. No calculator.

Find the derivative. Do not leave negative exponents or complex fractions in your answers.

1. \( f(x) = 3x^4 + \frac{2}{x} - 8x^{3/4} - 5 \)
2. \( y = 3x^2 \cos(5x) \)
3. \( f(x) = \frac{\tan x}{x^3} \)
4. \( y = \left(2x^3 + 5\right)^4 \)
5. \( f(x) = \sin\left(x^2\right) \)
6. \( y = \sin^3(5x) \)

Evaluate the given integrals.

7. \( \int \left(6x^2 - 5 + \frac{7}{x^2}\right) \, dx \)
8. \( \int (3x + 1)(2x - 5) \, dx \)
9. \( \int (5 + \sec x \tan x - \sec^2 x) \, dx \)
10. \( \int \frac{x - 4}{\sqrt{x^2 - 8x + 1}} \, dx \)
11. \( \int x^2 \cos(2x^3) \, dx \)
12. \( \int \sin^3(5x) \cos(5x) \, dx \)
13. \( \int_{\pi/12}^{\pi/2} \sin(3x) \, dx \)
14. \( \int_{1}^{2} x\left(x^2 + 1\right)^3 \, dx \)

Evaluate.

15. \( \frac{d}{dx} \int_{5}^{x} \sin\left(t^2\right) \, dt \)
16. \( \frac{d}{dx} \int_{0}^{\pi} \frac{1}{t^2 + 4} \, dt \)
17. \( \frac{d}{dx} \int_{x}^{3} e^{t^2} \, dt \)
18. \( \frac{d}{dx} \int_{3}^{\cos x} \left(t^2 - 25\right)^3 \, dt \)
19. \( \frac{d}{dx} \int_{-4}^{x} \sec \, dt \)
20. \( \frac{d}{dx} \int_{-2}^{\tan x} \cos\left(t^3\right) \, dt \)
CALCULUS AB
WORKSHEET ON SECOND FUNDAMENTAL THEOREM
AND FUNCTIONS DEFINED BY INTEGRALS

1. Evaluate.
   (a) \( \frac{d}{dx} \int_3^x \frac{\sin t}{t} \, dt \)
   (b) \( \frac{d}{dx} \int_\pi^x e^{-t^2} \, dt \)
   (c) \( \frac{d}{dx} \int_1^x \frac{1}{t} \, dt \)
   (d) \( \frac{d}{dx} \int_x^2 \ln(t^2) \, dt \)
   (e) \( \frac{d}{dx} \int_{-5}^x \cos(t^3) \, dt \)
   (f) \( \frac{d}{dx} \int_{\tan x}^{17} \sin(t^4) \, dt \)

2. The graph of a function \( f \) consists of a semicircle and two line segments as shown. Let \( g \) be the function given by \( g(x) = \int_0^x f(t) \, dt \).
   (a) Find \( g(0), g(3), g(-2), \) and \( g(5) \).
   (b) Find all values of \( x \) on the open interval \((-2, 5)\) at which \( g \) has a relative maximum. Justify your answers.
   (c) Find the absolute minimum value of \( g \) on the closed interval \([-2, 5]\) and the value of \( x \) at which it occurs. Justify your answer.
   (d) Write an equation for the line tangent to the graph of \( g \) at \( x = 3 \).
   (e) Find the \( x \)-coordinate of each point of inflection of the graph of \( g \) on the open interval \((-2, 5)\). Justify.
   (f) Find the range of \( g \).

3. Let \( g(x) = \int_0^x f(t) \, dt \), where \( f \) is the function whose graph is shown.
   (a) Evaluate \( g(0), g(1), g(2), g(3), \) and \( g(7) \).
   (b) Write an equation for the line tangent to the graph of \( g \) at \( x = 4 \).
   (c) On what interval(s) is \( g \) increasing? Decreasing? Justify your answer.
   (d) For what value of \( x \) does the graph of \( g \) have a relative maximum? Justify your answer.
   (e) For what value of \( x \) does the graph of \( g \) have its absolute maximum value? What is the maximum value? Justify your answer.
   (f) For what value of \( x \) does the graph of \( g \) have its absolute minimum value? What is the minimum value? Justify your answer.

4. Let \( g(x) = \int_0^x f(t) \, dt \), where \( f \) is the function whose graph is shown.
   (a) On what intervals is \( g \) decreasing? Justify.
   (b) For what value(s) of \( x \) does \( g \) have a relative maximum? Justify.
   (c) On what intervals is \( g \) concave down? Justify.
   (d) At what values of \( x \) does \( g \) have an inflection point? Justify.

Evaluate the given integrals.

5. \( \int_0^x \frac{x^2 + 2x - 3}{x^4} \, dx \)
6. \( \int_0^x \frac{1}{\sqrt[3]{4} + 2} \, dx \)
7. \( \int_0^4 \frac{x}{\sqrt{9 + x^2}} \, dx \)
8. \( \int_0^\sqrt{3} \cos(x^3) \, dx \)
9. \( \int_{-\pi/12}^{\pi/6} \sin(2x) \, dx \)
10. \( \int_{-\pi/12}^{\pi/6} \cos^5(x^3) \sin(x^3) \, dx \)
CALCULUS AB
WORKSHEET 2 ON FUNCTIONS DEFINED BY INTEGRALS

Work the following on notebook paper.

1. Find the equation of the tangent line to the curve \( y = F(x) \) where \( F(x) = \int_1^x \sqrt[3]{t^2 + 7} \, dt \) at the point on the curve where \( x = 1 \).

2. Suppose that \( 5x^3 + 40 = \int_c^x f(t) \, dt \).
   (a) What is \( f(x) \)?
   (b) Find the value of \( c \).

3. If \( F(x) = \int_{-4}^x (t-1)^2(t+3) \, dt \), for what values of \( x \) is \( F \) decreasing? Justify your answer.

4. Let \( H(x) = \int_0^x f(t) \, dt \) where \( f \) is the continuous function with domain \([0, 12]\) shown on the right.
   (a) Find \( H(0) \).
   (b) On what interval(s) of \( x \) is \( H \) increasing? Justify your answer.
   (c) On what interval(s) of \( x \) is \( H \) concave up? Justify your answer.
   (d) Is \( H(12) \) positive or negative? Explain.
   (e) For what value of \( x \) does \( H \) achieve its maximum value? Explain.

5. The graph of a function \( f \) consists of a semicircle and two line segments as shown on the right.
   Let \( g(x) = \int_1^x f(t) \, dt \).
   (a) Find \( g(1) \), \( g(3) \), \( g(-1) \).
   (b) On what interval(s) of \( x \) is \( g \) decreasing? Justify your answer.
   (c) Find all values of \( x \) on the open interval \((-3, 4)\) at which \( g \) has a relative minimum. Justify your answer.
   (d) Find the absolute maximum value of \( g \) on the interval \([-3, 4]\) and the value of \( x \) at which it occurs. Justify your answer.
   (e) On what interval(s) of \( x \) is \( g \) concave up? Justify your answer.
   (f) For what value(s) of \( x \) does the graph of \( g \) have an inflection point? Justify your answer.
   (g) Write an equation for the line tangent to the graph of \( g \) at \( x = -1 \).

6. The graph of the function \( f \), consisting of three line segments, is shown on the right.
   Let \( g(x) = \int_1^x f(t) \, dt \).
   (a) Find \( g(2) \), \( g(4) \), \( g(-2) \).
   (b) Find \( g'(0) \) and \( g'(3) \).
   (c) Find the instantaneous rate of change of \( g \) with respect to \( x \) at \( x = 2 \).
   (d) Find the absolute maximum value of \( g \) on the interval \([-2, 4]\). Justify.
   (e) The second derivative of \( g \) is not defined at \( x = 1 \) and at \( x = 2 \). Which of these values are \( x \)-coordinates of points of inflection of the graph of \( g \)? Justify your answer.
7. Given \( \int_{-1}^{3} f(x) \, dx = 9 \), find \( \int_{-1}^{3} (2f(x)+5) \, dx \).

8. Given \( f(x) = \begin{cases} 4x, & x \leq 1 \\ 4, & x > 1 \end{cases} \). Evaluate: \( \int_{1/2}^{5} f(x) \, dx \).

9. Find \( \frac{dy}{dx} \) given \( x^2 + 3xy + y^3 = 10 \).

10. A tank contains 120 gallons of oil at time \( t = 0 \) hours. Oil is being pumped into the tank at a rate \( R(t) \), where \( R(t) \) is measured in gallons per hour and \( t \) is measured in hours. Select values of \( R(t) \) are given in the table below.

<table>
<thead>
<tr>
<th>( t ) (hours)</th>
<th>0</th>
<th>3</th>
<th>5</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(t) ) (gallons per hour)</td>
<td>8.9</td>
<td>6.8</td>
<td>6.4</td>
<td>5.9</td>
<td>5.7</td>
</tr>
</tbody>
</table>

(a) Estimate the number of gallons of oil in the tank at \( t = 12 \) hours by using a **trapezoidal approximation** with four subintervals and values from the table. Show the computations that lead to your answer.

(b) Estimate the number of gallons of oil in the tank at \( t = 12 \) hours by using a **right Riemann sum** with four subintervals and values from the table. Show the computations that lead to your answer.

(c) A model for the rate at which oil is being pumped into the tank is given by the function

\[
G(t) = 3 + \frac{10}{1 + \ln(t+2)}, \quad \text{where } G(t) \text{ is measured in gallons per hour and } t \text{ is measured in hours.}
\]

Use the model to find the number of gallons of oil in the tank at \( t = 12 \) hours.
CALCULUS AB
WORKSHEET 3 ON FUNCTIONS DEFINED BY INTEGRALS

Work the following on notebook paper:

1. The function \( g \) is defined on the interval \([0, 6]\) by
   \[ g(x) = \int_0^x f(t) \, dt \]
   where \( f \) is the function graphed in the figure.
   (a) For what values of \( x \), \( 0 < x < 6 \), does \( g \) have a relative maximum? Justify your answer.
   (b) For what values of \( x \) is the graph of \( g \) concave down? Justify your answer.
   (c) Write an equation for the tangent line to \( g \) at the point where \( x = 3 \).
   (d) Sketch a graph of the function \( g \). List the coordinates of all critical point and inflection points.

2. Suppose that \( f' \) is a continuous function, that \( f(1) = 13 \), and that \( f(10) = 7 \). Find the average value of \( f' \) over the interval \([1, 10]\).

3. The graph of a differentiable function \( f \) on the closed interval \([-4, 4]\) is shown.
   Let \( G(x) = \int_{-4}^x f(t) \, dt \) for \(-4 \leq x \leq 4\).
   (a) Find \( G(-4) \).
   (b) Find \( G'(-4) \).
   (c) On which interval or intervals is the graph of \( G \) decreasing? Justify your answer.
   (d) On which interval or intervals is the graph of \( G \) concave down? Justify your answer.
   (e) For what values of \( x \) does \( G \) have an inflection point? Justify your answer.

4. The function \( F \) is defined for all \( x \) by \( F(x) = \int_0^x \sqrt{t^2 + 8} \, dt \). Find:
   (a) \( F'(x) \)
   (b) \( F'(1) \)
   (c) \( F''(x) \)
   (d) \( F''(1) \)

5. The function \( F \) is defined for all \( x \) by \( F(x) = \int_0^x f(t) \, dt \),
   where \( f \) is the function graphed in the figure. The graph of \( f \) is made up of straight lines and a semicircle.
   (a) For what values of \( x \) is \( F \) decreasing? Justify your answer.
   (b) For what values of \( x \) does \( F \) have a local maximum? A local minimum? Justify your answer.
   (c) Evaluate \( F(2), F'(2), \) and \( F''(2) \).
   (d) Write an equation of the line tangent to the graph of \( F \) at \( x = 4 \).
   (e) For what values of \( x \) does \( F \) have an inflection point? Justify your answer.

TURN->>>
6. If \( F(x) = \int_x^{-5} (t^2 - t - 6) dt \), on what intervals is \( F \) decreasing?

Use your calculator or problems 7 – 9.

7. \( f'(x) = \cos(x^2) \) and \( f(3) = 4.5 \). Find \( f(2) \).

8. Water is pumped out of a holding tank at a rate of \( 3 - 3e^{-0.15t} \) liters/minute, where \( t \) is in minutes since the pump is started. If the holding tank contains 2000 liters of water when the pump is started, how much water does it hold one hour later?

9. In a certain city, the temperature, in °F, \( t \) hours after 6 AM was modeled by the function

\[
T(t) = 53 + 15 \sin\left(\frac{\pi t}{12}\right).
\]

Find the average temperature during the period from 6 AM to 6 PM.

Remember the formula for average value: \( f_{AVE} = \frac{1}{b-a} \int_a^b f(x) dx \)

Evaluate.

10. \( \frac{d}{dx} \int_3^x \sin(t^2) dt \) 

11. \( \frac{d}{dx} \int_2^{\tan(x)} \frac{1}{t} dt \)
CALCULUS AB
WORKSHEET ON 5.1 – 5.2

Work the following on notebook paper. Do not use your calculator.

Find the derivative.

1. \( f(x) = \ln(x^3 + 5x) \)
2. \( y = x^4 \ln x \)
3. \( g(x) = \ln \sqrt{x^2 + 3} \)
4. \( y = \ln \left( \frac{4x}{x^2 + 5} \right) \)
5. \( f(x) = \int_3^{\ln(3x)} (2t + 5) \, dt \)

6. Write the equation of the tangent line to the graph of \( f(x) = 2x^3 + \ln(x + 2) \) at the point where \( x = 1 \).

7. Find \( \frac{dy}{dx} \) given \( 3x^2 + \ln(xy) - y^4 = 2 \).

Evaluate.

8. \( \int \frac{1}{2x+5} \, dx \)
9. \( \int (x^3 + 1)^5 \, x^2 \, dx \)
10. \( \int \frac{x}{x^2 + 4} \, dx \)
11. \( \int \frac{x}{\sqrt{x^2 + 4}} \, dx \)
12. \( \int x \sin(3x^2) \, dx \)
13. \( \int \frac{\cos x}{\sqrt{\sin x}} \, dx \)
14. \( \int \frac{\sin x}{1 + \cos x} \, dx \)
15. \( \int_0^4 \frac{2x}{\sqrt{x^2 + 9}} \, dx \)
16. \( \int x^2 \tan(x^3) \, dx \)
17. \( \int_0^{\pi/2} \sin^3 x \cos x \, dx \)
18. \( \int_1^e \frac{\ln x}{x} \, dx \)
19. \( \int_0^2 \frac{x^2 - 2}{x + 1} \, dx \)
20. \( \int_0^{\pi/2} \cos \left( \frac{2x}{3} \right) \, dx \)
21. \( \int_2^6 \frac{x - 4}{x + 1} \, dx \)
22. \( \int e^x \frac{1}{x \ln x} \, dx \)
23. \( \int \sec(3x) \, dx \)
CALCULUS AB
REVIEW WORKSHEET ON 1st & 2nd FUND. TH., AVE. VALUE, FUNCTIONS DEFINED BY INTEGRALS, & 5.1 – 5.2

Work the following on notebook paper.
Evaluate. No calculator.
1. \( \frac{d}{dx} \int_5^x \frac{3}{7+t^2} \, dt \)
2. \( \frac{d}{dx} \int_\pi^x \frac{5}{2+t} \, dt \)

3. The graph of a function \( f \) consists of a quarter circle and three line segments as shown. Let \( g \) be the function given by \( g(x) = \int_{-2}^x f(t) \, dt \).
(a) Find \( g(-4), g(-2), \) and \( g(7) \).
(b) Find \( g(4), g'(4), \) and \( g''(4) \).
(c) Find all values of \( x \) on the open interval \((-4,7)\) at which \( g \) has a relative maximum. Justify your answers.
(d) Find the absolute minimum value of \( g \) on the closed interval \([-4,7]\) and the value of \( x \) at which it occurs. Justify your answer.
(e) Find the \( x \)-coordinate of each point of inflection of the graph of \( g \) on the open interval \((-4,7)\). Justify your answer.

4. Given \( f(x) = \int_1^x (t^2 - 5t + 6) \, dt \).
(a) Write the equation of the tangent line to \( f \) at the point where \( x = 1 \).
(b) For what values of \( x \) is the graph of \( f \) increasing? Decreasing? Justify your answer.
(c) Find all values of \( x \) at which the graph of \( f \) has a relative maximum or a relative minimum. Justify your answers.
(d) On what intervals is the graph of \( f \) concave up? Concave down? Justify your answer.
(e) Find the \( x \)-coordinate of each point of inflection of the graph of \( f \). Justify your answer.

Find the derivative.
5. \( y = \ln \left( x^2 + 1 \right) \)
6. \( f(x) = \ln \frac{3x^2 + 1}{x^2 + 1} \)
7. \( y = x^3 \ln x \)
8. \( g(x) = \int_5^{\ln(2x)} (3t + 4) \, dt \)

9. Given \( x^3 + 5 \ln y + y^3 = 2 \), find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \).

Evaluate.
10. \( \int_0^3 \frac{1}{3x + 4} \, dx = \)
11. \( \int_0^3 \frac{x^2}{c} \left( \ln x \right)^4 \, dx = \)
12. \( \int_3^5 \frac{x^2 - 5}{x + 2} \, dx = \)
13. \( \int \tan(3x) \, dx = \)
14. \( \int x^2 \sec(x^3) \, dx = \)
15. \( \int \frac{x^2 + 4}{x} \, dx = \)
16. \( \int x^2 \cos \left( 2x^3 \right) \, dx = \)
17. \( \int_{\pi/8}^{\pi/6} \cos^2(2x) \sin(2x) \, dx = \)

TURN->>>
Use your calculator on problems 18 – 20. Give decimal answers correct to three decimal places.

18. $f'(x) = \sin(x^2)$ and $f(1) = -3$. Find $f(2)$.

19. $f'(x) = e^{-x^2}$ and $f(3) = -1$. Find $f(2)$.

20. A cup of coffee at 93°C is put into a 20°C room when $t = 0$. The coffee’s temperature is dropping at a rate of $r(t) = 5e^{-0.3t}$°C per minute, with $t$ in minutes. Find the coffee’s temperature when $t = 8$ minutes.

21. Given $f(x) = \begin{cases} x+1, & x \leq 0 \\ x^2 + 1, & x > 0 \end{cases}$. Evaluate $\int_{-3}^{2} f(x)dx$. (No calculator.)

22. Given $\int_{2}^{7} f(x)dx = 4$, find $\int_{2}^{7} (3f(x)+2)dx$. (No calculator.)